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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2451

MATHEMATICAL IMPROVEMENT OF METHOD FOR COMPUTING POISSON INTEGRALS INVOLVED IN DETERMINATION OF VELOCITY DISTRIBUTION ON AIRFOILS

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MATHEMATICAL IMPROVEMENT OF METHOD FOR COMPUTING POISSON INTEGRALS INVOLVED IN DETERMINATION OF VELOCITY DISTRIBUTION ON AIRFOILS

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SUMMARY

The Poisson integral involved in the determination of the change in velocity distribution resulting from a change in airfoil profile in parallel incompressible flow is solved.

First, three well-developed numerical methods of evaluating this integral, all based on the division of the range of integration into small equal intervals, and the difficulties involved in each method, are discussed. Then a new method, based on the use of unequal intervals, is developed, and compared with the other methods by means of several examples. The new method is found to give good results for both the direct and inverse airfoil problems and is easily adaptable to rather complicated problems. It is particularly recommended for all those functions where steep slopes in small portions of the region to be integrated exist.

INTRODUCTION

The ordinary thin airfoil at small angles of attack produces only slight disturbances in the flow of a parallel incompressible fluid. Hence, the influences of camber and thickness upon the velocity distribution may be computed independently and their effects superimposed. The effect of camber may be represented by vortices distributed along the chord line of the airfoil section; the effect of the thickness, by sources and sinks also along the same chord line. The velocities produced by these singularity distributions enable one to compute the pressure distribution on the airfoil rather quickly.

Allen (reference 1) has presented this singularity method in a form which has proved to be very practical for common usage. However, in special cases the unavoidable evaluation of the Poisson integral in

the course of the computations has given rise to numerical difficulties. Such integrals are usually computed by the application of finite differences using intervals of equal length. However, changes in airfoil shape, which result in marked changes in the function to be integrated in only small portions of the range of integration, require that extremely small interval sizes be employed in this range, and, consequently over the entire range of integration. This leads to a considerable amount of computational work; hence, it appears reasonable to discuss the possibility of employing intervals of varying lengths for the evaluation of the Poisson integral.

This investigation was prompted by the difficulties arising from the problem of small changes in the shape of symmetrical airfoils at the angle of zero lift. The examples included in the present report are restricted to this case, but the results obtained are in no way specialized and may be applied to all problems wherein the Poisson integral occurs.

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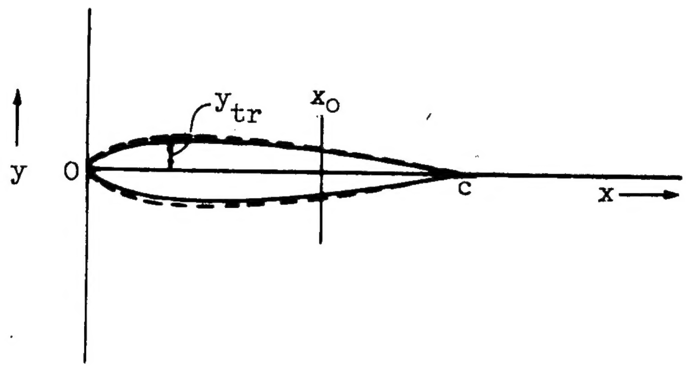
The author wishes to express her appreciation to Mr. H. Norman Abramson for his intelligent and skillful help in the computational work and for his assistance in writing the final report. The author also wishes to extend her thanks to Mr. R. E. Dannenberg and the computing staff of the 7- by 10-foot wind-tunnel section of the Ames Aeronautical Laboratory, Moffett Field, California, for preparing the extended tables of the functions j_{no} and j_{no}^*.

DISCUSSION OF PROBLEM

The basic reference profile may be given by $y_{tr} = f(x)$, and its velocity distribution may be known from an earlier computation. The problem at hand is that of determining the change in the velocity distribution resulting from a change in the shape of the profile (indicated by the dotted line in the following fig.). The difference of these two shapes is designated as Δy_t.



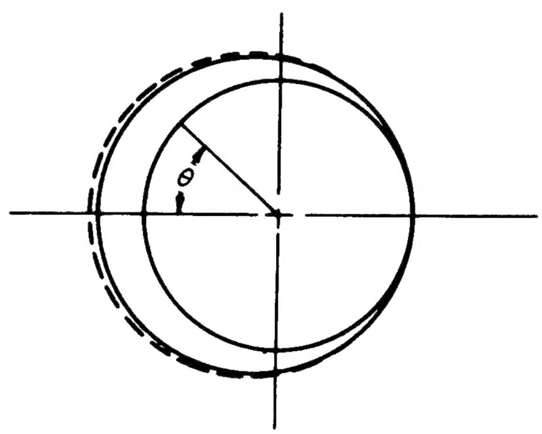
Allen (reference 1, p. 7) gives for the change of velocity the equation

$$\frac{\Delta v(x_o)}{V_o} = -\frac{1}{\pi}\int_0^c \frac{\frac{d(\Delta y_t)}{dx}\,dx}{x - x_o} \qquad (1)$$

where V_o is the velocity of the basic parallel flow. If, by conformal mapping of the outside flow region, the center line of the profile is transformed into a circle by the relation

$$x = \frac{c}{2}\,(1 - \cos\,\theta) \qquad (2)$$

then the profile is transformed into a curve approximating the circle shown below.



The change in velocity due to a change in form will then be given as

$$\frac{\Delta v}{V_o}\left(\theta_o\right) = -\frac{1}{2\pi}\int_0^{2\pi} \frac{d\left(\Delta y_t\right)}{dx} \cot \frac{\theta - \theta_o}{2}\, d\theta \tag{3}$$

defining

$$\left[\frac{d\left(\Delta y_t\right)}{dx}\right]_{\pi+\theta} = -\left[\frac{d\left(\Delta y_t\right)}{dx}\right]_{\pi-\theta}$$

This is the form most often used for computation purposes because the inverse problem (that of computing the change in shape due to a change in velocity distribution) utilizes the analytic form

$$\left[\frac{d\left(\Delta y_t\right)}{dx}\right]_{x_o} = \frac{1}{2\pi}\int_0^{2\pi} \frac{\Delta v}{V_o} \cot\left(\frac{\theta - \theta_o}{2}\right) d\theta \tag{4}$$

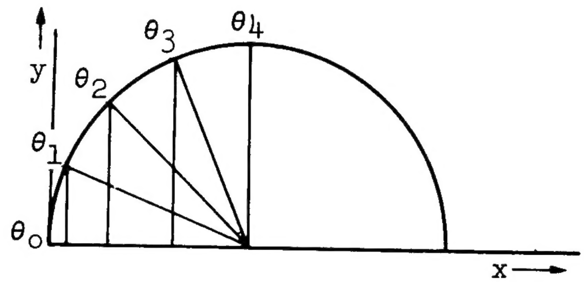
defining

$$\left(\frac{\Delta v}{V_o}\right)_{\pi+\theta} = \left(\frac{\Delta v}{V_o}\right)_{\pi-\theta}$$

which is strikingly similar. The corresponding formula in the original x,y coordinate system is given by

$$\left[\frac{d\left(\Delta y_t\right)}{dx}\right]_{x_o} = \frac{1}{\pi}\int_0^{c} \frac{\frac{\Delta v}{V_o}}{x - x_o} \frac{\sqrt{x_o\left(c - x_o\right)}}{\sqrt{x(c - x)}}\, dx \tag{5}$$

The evaluation of equation (3) may be accomplished by any one of several different methods; however, all of these methods employ the device of replacing the integral over the range 0 to 2π by a sum of integrals over intervals of equal length $\Delta\theta$. The equally distributed points θ_n have corresponding values x_n which are not equally distributed (see following fig.).

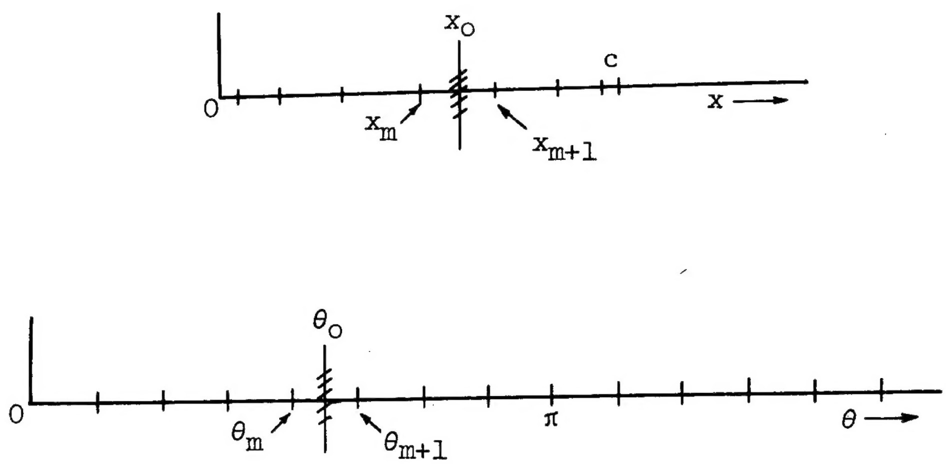


This arrangement is sometimes favorable, and sometimes not, depending upon the particular form of $\frac{d(\Delta y_t)}{dx}$. (See discussion following equation (43).)

The use of the angular coordinate θ has the advantage that the functions $\frac{d(\Delta y_t)}{dx}$ or $\frac{\Delta v}{V_o}$ are periodic functions in θ, and this periodicity facilitates the organization of the numerical computations. The disadvantage arises from the fact that these functions are usually given as functions of x, and, since the analytic form is not usually known, any transformations made will lead to small errors. For example, if $\frac{d(\Delta y_t)}{dx}$ or $\frac{\Delta v}{V_o}$ is given at special points which do not correspond to $\theta_n = n\Delta\theta$, then the computor must obtain the values of these functions for the values θ_1, θ_2, and so forth by interpolation.

DISCUSSION OF SOME OF THE EXISTING NUMERICAL SOLUTIONS OF POISSON INTEGRAL

The difficulty encountered in the solution of the Poisson integral arises from the fact that the term $\cot\frac{(\theta - \theta_o)}{2}$ or $\frac{1}{x - x_o}$ (equations (1) and (3), e.g.) approaches infinity when θ approaches θ_o or when x approaches x_o. The difficulty is of much less consequence when the function $\frac{d(\Delta y_t)}{dx}$ or $\frac{\Delta v}{V_o}$ is given analytically than when a numerical computation is undertaken. As a consequence, any simple integration, performed by replacing the integral with a summation over smaller intervals, always requires that the interval in which θ_o or x_o is located be given special consideration (see following fig.).



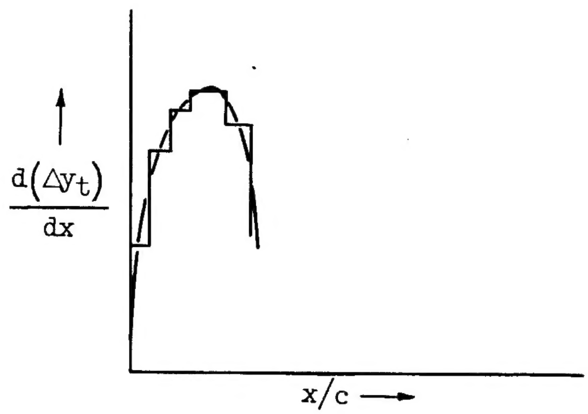
A majority of the solutions currently in use have been developed to such an extent that, for example, $\frac{\Delta v}{V_o}(\theta_o)$ is given by a sum of products of single values of $\left(\frac{d(\Delta y_t)}{dx}\right)_n$ and known factors A_n; that is,

$$\begin{aligned}\frac{\Delta v}{V_o}(\theta_o) &= -\frac{1}{2\pi}\int_0^{2\pi} \frac{d(\Delta y_t)}{dx} \cot\left(\frac{\theta - \theta_o}{2}\right) d\theta \\ &= -\frac{1}{2\pi}\int_{-\theta_o}^{2\pi-\theta_o} \frac{d(\Delta y_t)}{dx} \cot\frac{\theta^*}{2}\, d\theta^* \\ &= -\frac{1}{2\pi}\sum_n \int_{\theta_n{}^*}^{\theta_{n+1}{}^*} \frac{d(\Delta y_t)}{dx} \cot\frac{\theta^*}{2}\, d\theta^* \qquad (6)\end{aligned}$$

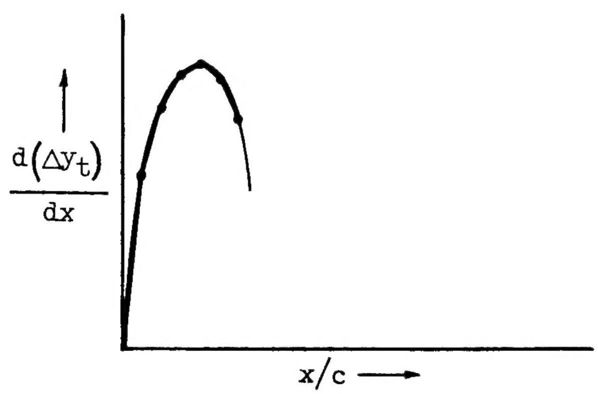
which leads to (see reference 2, e.g.)

$$\frac{\Delta v}{V_o}(\theta_o) = \sum_n A_{no}\left[\frac{d(\Delta y_t)}{dx}\right]_n \qquad (7)$$

The coefficients A_{no} depend upon the particular method of numerical integration which is employed. If, for example, $\frac{d(\Delta y_t)}{dx}$ is replaced by a step-curve, that is, assumed constant in every interval (see fig. below), one set of values of A_{no} would be obtained.



Greater accuracy would be obtained by the assumption that $\frac{d(\Delta y_t)}{dx}$ is replaced by straight-line segments (see fig. below), in which case a second set of values of A_{no} would be obtained.



A further refinement would be that of assuming $\frac{d(\Delta y_t)}{dx}$ to be composed of segments of parabolas, and so forth. Since the accuracy of the resulting values of $\frac{\Delta v}{V_o}$ depends upon both the character of the approximate curve and the size of interval taken, it is apparent that the same degree of accuracy might be achieved from many different combinations of interval sizes and approximations to the function $\frac{d(\Delta y_t)}{dx}$.

Naiman (reference 2) has used Simpson's rule for computing the Poisson integral, which corresponds to the replacement of the product $\left[\frac{d(\Delta y_t)}{dx} \cot \frac{\theta^*}{2}\right]$ by segments of parabolas. The "critical interval" $\left(\text{i.e., where } \cot \frac{\theta^*}{2} \longrightarrow \infty\right)$ was carefully treated by using differences of higher order $\left(\text{including the fifth derivative of } \frac{d(\Delta y_t)}{dx}\right)$. Naiman divided the period of 2π in 20 or 40 intervals and calculated the corresponding sets of values of A_{no}. Other workers at the NACA have extended the calculation of these values of A_{no} to 80 and 160 intervals (unpublished information).[1]

Obviously, the time required for computing $\frac{\Delta v}{V_o}$ increases with the number of intervals taken because of the increased number of multiplications to be performed. In addition, greater preparations for the computing process are necessarily involved, particularly since the values of $\frac{d(\Delta y_t)}{dx}$ needed must usually be obtained by interpolation. This interpolation has to be done rather carefully as it is often not sufficient simply to take the values of the plotted curve of $\frac{d(\Delta y_t)}{dx}$. This curve should be checked by difference tables if the values $\left[\frac{d(\Delta y_t)}{dx}\right]_n$ are to represent a smooth curve.

For those functions of Δy_t which may be well-represented by a Fourier series, there exists a simple method of evaluating the Poisson integral which has apparently been overlooked until the present time. This method has the advantage of leading to a computation which does not involve the derivative of Δy_t.

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[1]Naiman has also suggested a second method for computing the Poisson integral (see reference 3). In this second method he uses Fourier polynomials to represent the function $\frac{d(\Delta y_t)}{dx}$. The computing procedure is very simple; however, the results depend largely on the degree of approximation of $\frac{d(\Delta y_t)}{dx}$ by such a polynomial. Thus, for large families of functions results are good; however, cases are known to the author where results were not satisfactory because regions with steep gradients may not be represented well enough by a Fourier polynomial of moderate order.

Equation (6) may be written in a different form (reference 1, equation (43)) as follows:

$$\frac{\Delta v}{V_o} = \frac{1}{\pi}\int_0^{\pi} \frac{d(\Delta y_t)}{dx} \frac{\sin\theta}{\cos\theta - \cos\theta_o} d\theta \tag{8}$$

and this may be rewritten as

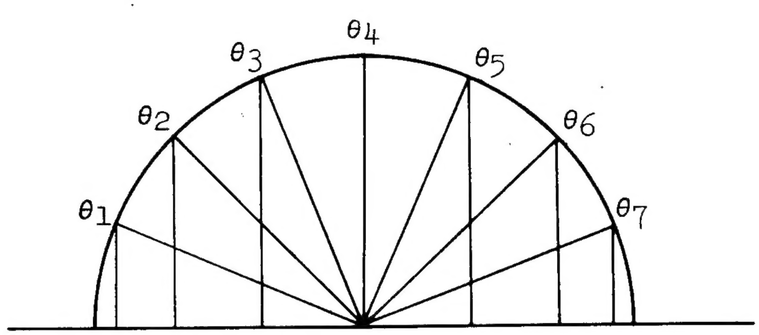
$$\frac{\Delta v}{V_o} = \frac{1}{\pi}\frac{2}{c}\int_0^{\pi} \frac{d(\Delta y_t)}{d\theta} \frac{d\theta}{\cos\theta - \cos\theta_o} \tag{9}$$

Equation (9) is strikingly similar to an integral ocurring in the theory of the lift distribution of a finite wing in incompressible flow. There, the induced angle α_i is given by

$$\alpha_i = \frac{1}{2\pi}\int_0^{\pi} \frac{d\gamma}{d\theta'} \frac{d\theta'}{\cos\theta' - \cos\theta} \tag{10}$$

where γ is the local dimensionless circulation.

Multhopp (reference 4) has given a solution for equation (10). He divides the range of integration into $(m_1 + 1)$ intervals (see fig. below)



with

$$\left.\begin{aligned} \theta_n &= \frac{n}{m_1 + 1}\pi \\ \gamma_n &= \gamma(\theta_n) \end{aligned}\right\} \tag{11}$$

and computes α_i at the points θ_n. He assumes that γ may be expanded in the form

$$\gamma = \sum C_\mu \sin \mu\theta$$

or

$$\gamma = \frac{2}{m + 1} \sum_{n=1}^{m_1} \gamma_n \sum_{\mu=1}^{m_1} \sin \mu\theta_n \sin \mu\theta \tag{12}$$

He then obtains the expression

$$\alpha_{i\nu} = b_{\nu\nu}\gamma_\nu - \sum_{1}^{m_1}{}' b_{\nu n}\gamma_n \tag{13}$$

The prime on the summation symbol indicates that $n = \nu$ is to be omitted from the summation because that special term has already been considered in the first term of the right-hand side (i.e., $b_{\nu\nu}\gamma_\nu$). Reference 4 presents tables for the coefficients $b_{\nu\nu}$ and $b_{\nu n}$ for $m_1 = 7$, 15, and 31. Applied to the problem at hand, $m_1 = 31$ would appear to be rather small; therefore a table for $m_1 = 63$ has been computed and is included in the present report (appendix A). As a comparison: For $m_1 = 63$, $\Delta\theta = 2.8125^\circ$; for Naiman's method with 160 points, $\Delta\theta = 2.25^\circ$.

Utilizing this method of integration which was developed by means of Fourier series, an expression may be obtained for the velocity distribution as follows:

$$\frac{\Delta v}{V_o}(\theta_o) = \frac{4}{c}\left(b_{\nu\nu}\,\Delta y_\nu - \sum_{1}^{m_1}{}' b_{\nu n}\,\Delta y_n\right) \tag{14}$$

The great advantage of this method is that of simplicity: (1) The actual computational procedure is very simple and (2) the derivative $\frac{d(\Delta y_t)}{dx}$ is avoided. The simplicity of computation is reflected in the fact that the time required for computing $\frac{\Delta v}{V_o}$ at one value of θ_o is approximately half that required by the method of Naiman when the intervals have approximately the same size. It should be noted,

however, that the accuracy of the method of Naiman will be greater than that of Multhopp in those cases where the differentiation of Δy_t by Fourier expansion (equation (12)) does not give good results.

A third method of evaluating the Poisson integral became known during the course of the present investigation. In a paper by Timman (reference 5), the integral is studied in the form

$$\tau(\phi) = -\frac{1}{2\pi}\int_0^{2\pi} \overline{\sigma}(\psi) \cot\frac{\phi - \psi}{2}\, d\psi \tag{15}$$

Timman assumes that $\overline{\sigma}(\psi)$ is not given analytically, but only at equidistant points. An interpolation polynomial (reference 6) for $\overline{\sigma}(\psi)$ is employed, and these polynomials replace the function $\overline{\sigma}(\psi)$ in a single interval by a function of third order. The polynomial function thus introduced has a continuous first derivative,[2] and it is evident that this continuity is essential for the attainment of good results.

Timman has divided the period 2π into 36 intervals of equal length and established a computing scheme. The function $\overline{\sigma}(\psi)$ is separated into its symmetrical and unsymmetrical parts so that

$$\overline{\sigma}(\psi) = s + d \tag{16}$$

Then

$$\tau(\psi_l) = \sum_{k=0}^{18} \alpha_{kl} s_k - \sum_{k=0}^{18} \beta_{kl}\, d_k \tag{17}$$

where the factors α_{kl} and β_{kl} are given in tabular form. In the present particular case $\frac{d(\Delta y_t)}{dx}$ is antisymmetrical (equation (3)) and $\frac{\Delta v}{V_o}$ is symmetrical (equation (4)). Thus the separation indicated by equation (16) does not require any additional work.

Timman's method should give good results provided that a sufficient number of intervals are taken - the division of 36 intervals over a period of 2π (i.e., 18 intervals over the chord of the profile) appears to be insufficient for an accurate representation of the function which occurs, $\frac{d(\Delta y_t)}{dx}$ or $\frac{\Delta v}{V_o}$.

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[2]The polynomials used in the classical interpolation formulas are less smooth (see reference 5, pp. 7 and 10, figs. 1 and 2).

The time required for computing one point by the method of Timman is approximately the same as for Naiman's method with the same interval size.

Other methods of evaluating the Poisson integral have been suggested. They will not be discussed here as it is the intention of this section to consider only the most practical of the known methods. The three methods already discussed have their own particular advantages and have been especially developed for rapid and simple computation; however, all three of these methods, when $\frac{d(\Delta y_t)}{dx}$ or $\frac{\Delta v}{V_o}$, change rapidly in magnitude, become cumbersome, and require that very small intervals be taken over the entire range of integration because the scheme of equal interval size is utilized.

EVALUATION OF POISSON INTEGRAL BY A METHOD EMPLOYING UNEQUAL INTERVALS

Development of Method

As the change in airfoil shape, or the change in velocity distribution, is given originally as a function of x it appears logical to retain the coordinate x in selecting the size of the different intervals. Hence, the Poisson integral may be studied in the form

$$\tau(x_o) = -\frac{1}{\pi}\int_0^c \sigma(x)\,\frac{dx}{x - x_o} \tag{18}$$

which corresponds to equation (1). Conforming with its physical meaning $\sigma(x) = \frac{d(\Delta y_t)}{dx}$ is assumed to be a function which is finite in every point of its range of definition.[3]

Define

$$\Delta x_n = x_{n+1} - x_n \tag{19a}$$

with

$$n = 0, 1, 2, 3, \ldots$$

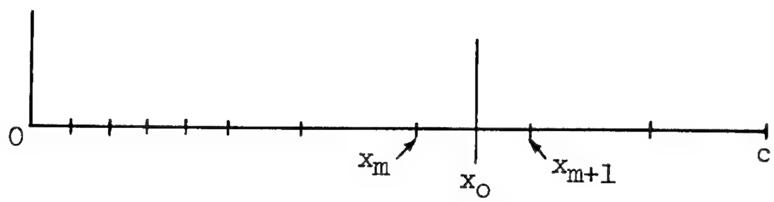
[3]This restriction will be dropped later; see discussion beginning with the first paragraph after equation (32).

and

$$x_m < x_o < x_{m+1} \tag{19b}$$

For convenience, there is chosen (see following fig.)

$$x_o = \frac{x_m + x_{m+1}}{2} \tag{19c}$$



The function $\sigma(x)$ is approximated by straight-line segments (see third sketch in preceding section). Then, for $x_n < x < x_{n+1}$,

$$\sigma(x) = \sigma(x_n) + \frac{\sigma(x_{n+1}) - \sigma(x_n)}{\Delta x_n}(x - x_n)$$

$$= \sigma_n + \frac{\sigma_{n+1} - \sigma_n}{\Delta x_n}(x - x_n) \tag{20}$$

from which there is obtained

$$\tau(x_o) = -\frac{1}{\pi}\int_0^c \sigma(x)\frac{dx}{x - x_o} = -\frac{1}{\pi}\sum\int_{x_n}^{x_{n+1}} \frac{\sigma_n + \frac{\sigma_{n+1} - \sigma_n}{\Delta x_n}(x - x_n)}{x - x_o}\,dx$$

$$= -\frac{1}{\pi}\left[\sum\int_{x_n}^{x_{n+1}} \frac{\sigma_n + \frac{\sigma_{n+1} - \sigma_n}{\Delta x_n}(x - x_o + x_o - x_n)}{x - x_o}\,dx\right]$$

$$= -\frac{1}{\pi}\left\{\sum\left(\frac{\sigma_{n+1} - \sigma_n}{\Delta x_n}\right)\Delta x_n + \sum\left[\sigma_n + \frac{\sigma_{n+1} - \sigma_n}{\Delta x_n}(x_o - x_n)\right]\left(\int_{x_n}^{x_{n+1}} \frac{dx}{x - x_o}\right)\right\} \tag{21}$$

Also,

$$\int_{x_n}^{x_{n+1}} \frac{dx}{x - x_o} = j_{no} \tag{22}$$

by definition. The function j_{no}, in the different regions of x, is given by different expressions as follows:

$$j_{no} = \left\{\begin{array}{ll} \log_e \frac{x_{n+1} - x_o}{x_n - x_o} & \text{for} \quad x_{n+1} > x_n > x_o \\ \log_e \frac{x_{n+1} - x_o}{x_o - x_n} & \text{for} \quad x_{n+1} > x_o > x_n \\ \log_e \frac{x_o - x_{n+1}}{x_o - x_n} & \text{for} \quad x_o > x_{n+1} > x_n \end{array}\right\} \tag{23}$$

Introducing j_{no} into equation (21), there results

$$\tau(x_o) = -\frac{1}{\pi}\left\{\sum(\sigma_{n+1} - \sigma_n) + \sum\left[\sigma_n + \frac{\sigma_{n+1} - \sigma_n}{\Delta x_n}(x_o - x_n)\right] j_{no}\right\}$$

$$= -\frac{1}{\pi}\left[\sum \sigma_n j_{no} + \sum(\sigma_{n+1} - \sigma_n)\left(1 + \frac{x_o - x_n}{\Delta x_n} j_{no}\right)\right] \tag{24}$$

Or, defining

$$1 + \frac{x_o - x_n}{\Delta x_n} j_{no} = j_{no}^{*} \tag{25}$$

there results, finally,

$$\tau(x_o) = -\frac{1}{\pi}\left[\sum \sigma_n j_{no} + \sum(\sigma_{n+1} - \sigma_n) j_{no}^{*}\right] \tag{26}$$

Since $x_{n+1} = x_n + \Delta x_n$, the functions j_{no} and j_{no}^{*} may be written as

$$\left.\begin{aligned}
j_{no}^{*} &= 1 + \frac{x_o - x_n}{\Delta x_n} j_{no} \\
j_{no} &= \log_e\left(1 + \frac{\Delta x_n}{x_n - x_o}\right) \quad \text{for} \quad x_n > x_o \\
&= \log_e\left(-1 + \frac{\Delta x_n}{x_o - x_n}\right) \quad \text{for} \quad x_n + \Delta x_n > x_o > x_n \\
&= \log_e\left(1 + \frac{\Delta x_n}{x_n - x_o}\right) \quad \text{for} \quad x_o > x_n + \Delta x_n
\end{aligned}\right\} \tag{27a}$$

and this form shows that j_{no} and j_{no}^{*} are functions of $\frac{x_n - x_o}{\Delta x_n}$ only.

$$\left.\begin{array}{llll} \text{For} & \dfrac{x_n - x_o}{\Delta x_n} \to \pm\infty & j_{no} \to 0 & j_{no}^* \to 0 \\ \text{For} & x_o - x_n = \frac{1}{2}\Delta x_n & j_{no} = 0 & j_{no}^* = 1 \end{array}\right\} \tag{27b}$$

For very large $\dfrac{x_n - x_o}{\Delta x_n} = \xi$,

$$\left.\begin{array}{l} j_{no} \to \dfrac{1}{\xi} - \dfrac{1}{2\xi^2} + \ldots \\ j_{no}^* \to \dfrac{1}{2\xi} - \dfrac{1}{3\xi^2} + \ldots \end{array}\right\} \tag{27c}$$

For very large negative $\dfrac{x_n - x_o}{\Delta x_n}$ with $\left|\dfrac{x_n - x_o}{\Delta x_n}\right| = \xi^*$,

$$\left.\begin{array}{l} j_{no} \to -\dfrac{1}{\xi^*} - \dfrac{1}{2\xi^{*2}} + \ldots \\ j_{no}^* \to -\dfrac{1}{2\xi^*} - \dfrac{1}{3\xi^{*2}} + \ldots \end{array}\right\} \tag{27d}$$

These functions are given in figure 1 and in table I.

It is seen that high absolute values of j_{no} and j_{no}^* occur near those values of $\dfrac{x_n - x_o}{\Delta x_n}$ which characterize the critical interval.[4]

Figure 1 gives an idea of the characteristic qualities of j_{no} and j_{no}^* as functions of $\dfrac{x_n - x_o}{\Delta x_n}$; however, the representation is not sufficient for picking out values for a computation. Table I gives the values of j_{no} and j_{no}^* for $-49.5 < \dfrac{x_n - x_o}{\Delta x} < 49.5$. This table might

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[4] If $x_o = \dfrac{x_m + x_{m+1}}{2}$ (equation (19c)) the critical interval is given by $-0.5 < \dfrac{x_n - x_o}{\Delta x_n} < 0.5$.

be used for rough computation and for getting acquainted with the method. In general, it is advisable to use those tables which are given in appendix B.

It will prove of benefit to investigate the exactness of that portion of the integral which contains the singularity. Recalling that the function $\sigma(x)$ was replaced by a straight line in every interval (equation (20)), there is obtained:

$$-\frac{1}{\pi}\int_{x_0-\frac{\Delta x}{2}}^{x_0+\frac{\Delta x}{2}} \frac{\sigma(x)}{x - x_0}\,dx = -\frac{1}{\pi}\left(\sigma_{m+1} - \sigma_m\right) \tag{28}$$

if $x_m < x_o < x_{m+1}$. Now, let an expansion of the function $\sigma(x)$ in the critical interval around x_o be assumed as follows:

$$\sigma(x) = \sigma(x_o) + \sigma'(x_o)(x - x_o) + \frac{\sigma''(x_o)}{2!}(x - x_o)^2 + \frac{\sigma'''(x_o)}{3!}(x - x_o)^3 + \frac{\sigma^{iv}(x_o)}{4!}(x - x_o)^4 + \ldots \tag{29}$$

Then,

$$\begin{aligned}-\frac{1}{\pi}\int_{x_o-\frac{\Delta x}{2}}^{x_o+\frac{\Delta x}{2}} \frac{\sigma(x)}{x - x_o}\,dx &= -\frac{1}{\pi}\left[\sigma'(x_o)\,\Delta x + \frac{\sigma'''(x_o)}{3!}\,\frac{2}{3}\left(\frac{\Delta x}{2}\right)^3 + \ldots\right] \\ &= -\frac{1}{\pi}\left[\frac{13}{12}\left(\sigma_{m+1} - \sigma_m\right) - \frac{1}{36}\left(\sigma_{m+2} - \sigma_{m-1}\right)\right] \\ &= -\frac{1}{\pi}\left\{\frac{19}{18}\left(\sigma_{m+1} - \sigma_m\right) - \frac{1}{36}\left[\left(\sigma_{m+2} - \sigma_{m+1}\right) + \left(\sigma_m - \sigma_{m-1}\right)\right]\right\}\end{aligned} \tag{30}$$

Comparison of formulas (30) and (28) shows that the error in the critical interval is approximately given by

$$- \frac{1}{\pi}\left\{\frac{1}{18}\left(\sigma_{m+1} - \sigma_m\right) - \frac{1}{36}\left[\left(\sigma_{m+2} - \sigma_{m+1}\right) + \left(\sigma_m - \sigma_{m-1}\right)\right]\right\} \tag{31}$$

The error of evaluating the whole integral by finite differences may be estimated by using two different interval distributions and comparing the results for a given x_o.

However, in addition to that error of the result produced by replacing the Poisson integral by a sum there exists another error. This sum cannot be computed exactly, but has a certain error depending on the accuracy of the given data for $\sigma_n(x)$ and the tabulated values of j_{no} and j_{no}^*. As the function $\sigma(x) = \frac{d(\Delta y_t)}{dx}$ usually has an error of $\epsilon_1 = 1 \times 10^{-3}$ it has proved amply satisfactory to give j_{no} and j_{no}^* to four decimal places, the error being less than $\epsilon_2 = 5 \times 10^{-5}$. The error of

$$\tau(x_o) = -\frac{1}{\pi}\left[\sum \sigma_n j_{no} + \sum \left(\sigma_{n+1} - \sigma_n\right) j_{no}^*\right]$$

is smaller than its upper limit given by

$$\frac{1}{\pi}\left[\epsilon_1\left(\sum \left|j_{no}\right| + 2\sum \left|j_{no}^*\right|\right) + \epsilon_2\left(\sum \left|\sigma_n\right| + \sum \left|\sigma_{n+1} - \sigma_n\right|\right)\right] \tag{32}$$

This formula shows that the influence of ϵ_1 is stronger than the influence of ϵ_2 as long as $\sum |\sigma_n| + \sum |\sigma_{n+1} - \sigma_n|$ is smaller than 1 - as it is in our later examples - and the sums $\sum |j_{no}|$ and $\sum |j_{no}^*|$ are always larger than 1. An increase of subdivisions makes the sums in the upper limit of the error (32) grow, thus requiring a higher accuracy, especially in σ_n and perhaps also in the values of j_{no} and j_{no}^*.

In establishing the solution of equation (18) it was assumed that $\sigma(x)$ is finite throughout its range of definition. If it is desired to compute the change in shape due to a proposed change of velocity distribution, this restriction must be eliminated, as will be recognized immediately.

Equation (5) may be written in the form

$$\frac{d(\Delta y_t)}{dx} = \frac{1}{\pi}\sqrt{x_o(c - x_o)} \int_0^c \frac{\Delta v/V_o}{\sqrt{x(c - x)}} \frac{dx}{x - x_o} \tag{5'}$$

Omitting the factor $\sqrt{x_o(c - x_o)}$, which does not affect the integration process, the integral may be reduced to the form of equation (18) by defining

$$\frac{\Delta v/V_o}{\sqrt{x(c - x)}} = \sigma_1(x) \tag{33}$$

However, $\sigma_1(x)$ will be infinite at $x = 0$ and $x = c$ if $\left(\frac{\Delta v}{V_o}\right)_o \neq 0$ and $\left(\frac{\Delta v}{V_o}\right)_c \neq 0$; therefore, a special consideration of the neighborhood of $x = 0$ and $x = c$ is required. This is done by splitting the integral into the following three parts:

$$\int_0^c \sigma_1(x)\frac{dx}{x - x_o} = \int_0^{\epsilon_1} \sigma_1(x)\frac{dx}{x - x_o} + \int_{\epsilon_1}^{c-\epsilon_2} \sigma_1(x)\frac{dx}{x - x_o} + \int_{c-\epsilon_2}^{c} \sigma_1(x)\frac{dx}{x - x_o} \tag{34}$$

with ϵ_1 and ϵ_2 being small compared with c. The integral

$$\int_{\epsilon_1}^{c-\epsilon_2} \sigma_1(x)\frac{dx}{x - x_o}$$

may be treated as was explained formerly for

$$\int_0^c \sigma(x)\frac{dx}{x - x_o}$$

(see equation (18)) because $\sigma_1(x)$ is finite for $\epsilon_1 < x < c - \epsilon_2$. For the first and third integrals, however, a new integration formula must be developed. By introducing

$$\mu = c - x \quad \text{and} \quad \sigma_1(x) = \sigma_1\left[\mu(x)\right] = \sigma_1{}^*(\mu)$$

there is obtained

$$\int_{c-\epsilon_2}^{c} \sigma_1(x)\frac{dx}{x - x_o} = -\int_0^{\epsilon_2} \sigma_1{}^*(\mu)\frac{d\mu}{\mu - \mu_o} \tag{35}$$

Hence, the method used for the first integral will also apply to the third. In most cases $\left(\frac{\Delta v}{V_o}\right)_c$ will be zero and there will be no need for a special evaluation in the neighborhood of $x = c$.

The integral

$$\int_0^{\epsilon_1} \sigma_1(x)\frac{dx}{x - x_o} = \int_0^{\epsilon_1} \frac{\Delta v/V_o}{\sqrt{x(c - x)}}\,\frac{dx}{x - x_o} \tag{36}$$

will have an important influence on the result of equation (34) only if x_o is near to ϵ_1. First the general formula will be given and then a simplification will be discussed for $x_o \gg \epsilon_1$.

The integral (36) will be solved assuming that

$$\frac{\Delta v}{V_o} = a_o + a_1\left(\frac{x}{c}\right) + a_2\left(\frac{x}{c}\right)^2 \quad \text{for} \quad 0 < x < \epsilon_1 \tag{37}$$

Only the final formula of this procedure is given here; the details of the solution will be found in appendix C.

$$F_1(x_o) = \int_0^{\epsilon_1} \frac{\Delta v/V_o}{\sqrt{x(c - x)}} \frac{dx}{x - x_o} = \frac{1}{c}\left\{M_o \frac{1}{\sqrt{\frac{x_o}{c}}}\left[a_o + a_1^*\left(\frac{x_o}{c}\right) + a_2^*\left(\frac{x_o}{c}\right)^2\right] + 2\sqrt{\frac{\epsilon_1}{c}}\left[a_1^* + a_2^*\left(\frac{x_o}{c}\right)\right] + \frac{2}{3} a_2^* \sqrt{\frac{\epsilon_1}{c}}^3\right\} \tag{38}$$

with M_o given in figure 2, and

$$\left.\begin{aligned}
a_o &= \left(\frac{\Delta v}{V_o}\right)_o \\
a_1^* &= a_1 + \frac{1}{2} a_o \\
\text{with} \quad a_1 &= \frac{c}{2\epsilon_1}\left[-3\left(\frac{\Delta v}{V_o}\right)_o + 4\left(\frac{\Delta v}{V_o}\right)_{\epsilon_1} - \left(\frac{\Delta v}{V_o}\right)_{2\epsilon_1}\right] \\
a_2^* &= a_2 + \frac{1}{2} a_1 + \frac{3}{8} a_o \\
\text{with} \quad a_2 &= \frac{c^2}{2\epsilon_1^{\,2}}\left[\left(\frac{\Delta v}{V_o}\right)_o - 2\left(\frac{\Delta v}{V_o}\right)_{\epsilon_1} + \left(\frac{\Delta v}{V_o}\right)_{2\epsilon_1}\right]
\end{aligned}\right\} \tag{39}$$

The coefficients a_o, a_1, and a_2 may be determined first, as they do not depend upon the particular value of x_o, and then $F_1(x_o)$ may be computed. The term depending on a_1 and a_2 will exert an influence only for small values of x_o/c. After a brief training the computor should be able to decide rather accurately when the formula

$$F_1(x_o) = \frac{1}{c} M_o \frac{a_o}{\sqrt{\frac{x_o}{c}}} \longrightarrow \frac{a_o}{x_o}\left(-2\sqrt{\frac{\epsilon_1}{c}}\right) \tag{40}$$

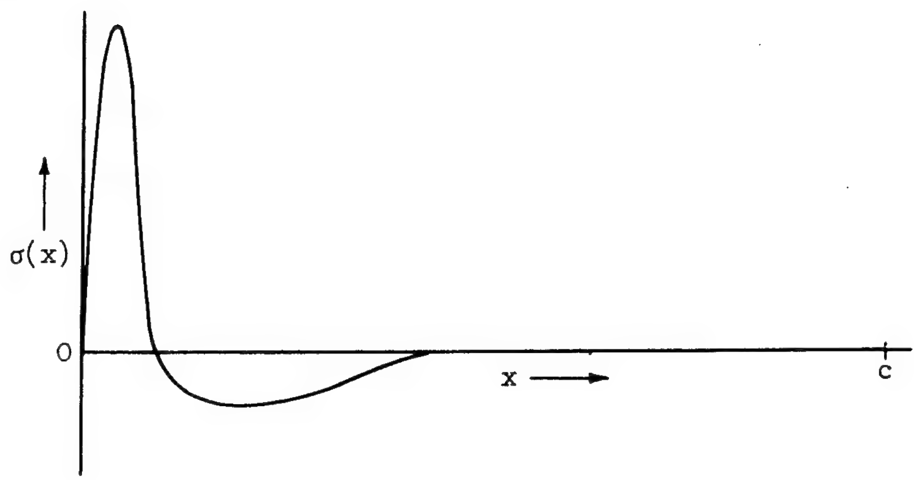
is sufficient and when the more exact expression (equation (38)) is required (also see fig. 2).

Organization of Computational Procedure for Unequal Intervals;

Transition from One Size of Interval to Another

A thorough understanding of the method is best achieved by following through a rather simple example; in addition various short cuts to the method will be demonstrated.

Assume a function $\sigma(x)$ of the type shown in the following figure.



It appears reasonable to take rather small intervals for small values of x because of the form of the curve $\sigma(x)$; therefore, the following arrangement of interval sizes is arbitrarily selected:

$$\overline{\Delta x} = 0.002 \quad \text{for} \quad 0 < x < 0.030$$

$$\overline{\overline{\Delta x}} = 0.006 \quad \text{for} \quad 0.030 < x < 0.096$$

Compute $\tau(0.009)$ with the help of equation (26). Note that the critical interval extends from 0.008 to 0.010. Table II(a) gives the values of x/c, σ_n, $\sigma_{n+1} - \sigma_n$, j_{no}, and j_{no}^* for the range with $\overline{\Delta x} = 0.002$. At $x = 0.030$ the interval changes to $\overline{\overline{\Delta x}} = 0.006$ and the same functions are given for the range with this size of interval in table II(b). Naturally the range above the broken line in table II(b) is not utilized in the computation since this portion has been considered in table II(a).

Note that $\frac{x_n - x_o}{\Delta x}$ progresses in table II(b) in the same manner as in table II(a); this is due to the special choice of $\overline{\overline{\Delta x}}$. If $\overline{\overline{\Delta x}} = 0.006$ were used starting with $x = 0$ the critical interval for $x_o = 0.009$ would extend from 0.006 to 0.012. Hence, for $\overline{\overline{\Delta x}} = 0.006$, $j_{no} = 0$ and $j_{no}^* = 1$ is to be found at $x/c = 0.006$.

For rapid computation it is best to have j_{no} and j_{no}^* as functions of $\frac{x_n - x_o}{\Delta x}$ on a paper strip and to place this strip adjacent to the columns headed by σ_n and $\sigma_{n+1} - \sigma_n$. If $\frac{x_n - x_o}{\Delta x}$ progresses as indicated in table I, the correct location of $j_{no} = 0$ and $j_{no}^* = 1$ at the beginning of the critical interval fixes the placement of the strip.

In the example just treated, the transition from one size of interval to another is very easy because x_o lies at the midpoint of an interval of the size 0.006 as well as of the size 0.002, if starting with $x = 0$.

If $\overline{\overline{\Delta x}}$ had been chosen 0.004, such a desirable arrangement would not have resulted because $x_o = 0.009$ would not be located at the midpoint of an interval of this size (starting with such intervals at $x = 0$).

As a second example compute the value of τ at $x_o = 0.015$. Again, $\frac{x_n - x_o}{\overline{\Delta x}}$ and $\frac{x_n - x_o}{\overline{\overline{\Delta x}}}$ will progress as in table I. The values $j_{no} = 0$ and $j_{no}^* = 1$ will be placed opposite $x/c = 0.014$ for the region with $\overline{\Delta x} = 0.002$ and opposite $x/c = 0.012$ for the region with $\overline{\overline{\Delta x}} = 0.006$. As long as $\overline{\overline{\Delta x}} = 3\overline{\Delta x}$, $\overline{\overline{\overline{\Delta x}}} = 3\overline{\overline{\Delta x}}$, and so forth and if x_o is chosen so as to be at the midpoint of the largest size of interval, the computation may be accomplished by shifting the strip with j_{no} and j_{no}^* corresponding to table I.

But suppose that the interval sizes are so arranged and it is desired to compute a point where x_o does not lie at the midpoint of the largest size of interval; for example, $x_o = 0.013$. The value $x_o = 0.013$ lies at the midpoint of an interval with $\overline{\Delta x} = 0.002$; hence,

for the range $0 < x < 0.030$, j_{no} and j_{no}^* may be taken directly from table I. However, at $x/c = 0.030$, intervals of the size $\overline{\overline{\Delta x}} = 0.006$ commence and there is obtained

$$\frac{x_n - x_o}{\overline{\overline{\Delta x}}} = \frac{0.030 - 0.013}{0.006} = 2.833$$

The value of $\frac{x_n - x_o}{\Delta x}$ progresses by 1, that is, 2.833, 3.833, 4.833, ... Thus the functions j_{no} and j_{no}^* are needed for values of $\frac{x_n - x_o}{\Delta x}$ which are not given in table I. One might think of taking them out of an enlarged diagram (see fig. 1); however, it is much more convenient to take them out of an extended table, which is conveniently arranged for "advancing by 1." Such tables are given in appendix B.

The example presented by the figure at the beginning of this section suggested starting at $x = 0$ with the smallest intervals. However, other examples may suggest another distribution of intervals. The smallest size of intervals may lie at any part of $0 < x < c$. There are no restrictions in the arrangement of intervals. (See, e.g., discussion following equation (43).)

Accuracy of Method, Examined by Means of an Analytical Example

The accuracy of the result depends directly upon the size of the interval taken and the reliability of the data comprising the function $\sigma(x)$. Because the function $\sigma(x)$ will be replaced by a broken line, a glance at the curve will quickly suggest an arrangement of intervals. In addition, the error in the critical interval may be used as a first test of the choice of intervals.

As a test of the quality of this new method, involving unequal intervals, a function $\sigma(x) = \frac{d(\Delta y_t)}{dx}$ has been treated which allows the analytical computation of $\tau(x) = \frac{\Delta v}{V_o}$.

The function $\sigma(x)$ is given analytically as

$$0 \leqq x \leqq 2\Delta \qquad \frac{d(\Delta y_t)}{dx} = Bx(2\Delta - x)$$

$$2\Delta \leqq x \leqq c_1 \qquad \frac{d(\Delta y_t)}{dx} = -D(c_1 - x)(x - 2\Delta)$$

$$c_1 \leqq x \leqq c \qquad \frac{d(\Delta y_t)}{dx} = 0$$

The following arbitrary values have been selected:

$$c_1 = 0.35$$

$$c = 1.0$$

D = Some multiple of B so that $\displaystyle\int_0^{c_1} \frac{d(\Delta y_t)}{dx}\, dx = 0$

The functions Δy_t and $\dfrac{d(\Delta y_t)}{dx}$ are given in figures 3(a) and 3(b), respectively.

The analytical computation of $\frac{\Delta v}{V_o}$ for figure 3(b) is given in figures 4(a) and 4(b). The arrangement of the unequal division for the numerical computation of $\frac{\Delta v}{V_o}$ is indicated in figure 4(a).[5]

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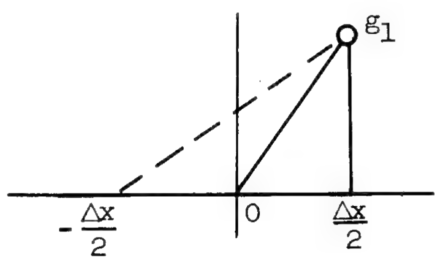
[5] It was desirable to obtain the value of $\frac{\Delta v}{V_o}$ at $x_o = 0$; hence, the first interval has been placed so that $-0.001 < x_o < 0.001$ and the function $\frac{d(\Delta y_t)}{dx} = 0$ for $-0.001 < x < 0$. Since the function $\frac{d(\Delta y_t)}{dx} = g$ is replaced in every interval by a straight line, the error might be expected to be large. However, $g = 0$ at $x_o = 0$ will aid in preventing the error from being too large.



A more exact solution would be obtained by putting

$$g = 0 \qquad -\frac{\Delta x}{2} < x < 0$$

$$g = \frac{g_1}{\frac{\Delta x}{2}} x \qquad 0 < x < \frac{\Delta x}{2}$$

$$-\frac{1}{\pi}\int_0^{\Delta x/2}\left(\frac{g_1}{\frac{\Delta x}{2}} x\right)\frac{dx}{x - x_o} = -\frac{1}{\pi} g_1 \times 1$$

For the interval $-\frac{\Delta x}{2} < x < \frac{\Delta x}{2}$ equation (26) would yield

$$-\frac{1}{\pi}\int_{-\Delta x/2}^{\Delta x/2} g \frac{dx}{x - x_o} = -\frac{1}{\pi}\left[\left(g_1 - 0\right) j_{no}^* + 0 \times j_{no}\right] = -\frac{1}{\pi} g_1$$

and no error is introduced. For $x_o \neq 0$ there is a very small error which may be avoided by respecting the change of size of the interval near $x = 0$.

Also given in figure 4(a) are points of the $\frac{\Delta v}{V_o}$ curve determined by the method of unequal intervals. Figure 4(b) presents the same information plotted to a larger scale.

For comparative purposes the same problem has been treated by the three methods of computation discussed earlier, namely, those of Naiman, Multhopp, and Timman. Figures 5(a) and 5(b) show the results obtained by the method of Naiman; obviously, the 40-point solution does not use a sufficiently accurate representation of the $\frac{d(\Delta y_t)}{dx}$ curve, while the 80- and 160-point solutions are quite good, with the exception of the maximum and minimum points of the $\frac{\Delta v}{V_o}$ curve. In order to obtain a value at approximately $\frac{x}{c} = 0.036$ a solution involving 320 points would be required. In this respect the method of unequal intervals is more adaptable to special conditions without involving much new work than is the method of Naiman.

The results obtained by Multhopp's method are given in figures 6(a) and 6(b). The 31-point solution (in Multhopp's somewhat odd manner of designation) corresponds to $\Delta\theta = 5.625^\circ$; the 63-point solution, to $\Delta\theta = 2.8125^\circ$. The computation is very simple and the results of the method with 63 points are comparable with that of Naiman with 80 points, with the exception of those near the region $0 < x < 0.01$ (this is shown most clearly in fig. 6(b)). The very steep peak of $\frac{d(\Delta y_t)}{dx}$ at $x/c = 0.02$ requires rather high harmonics for the representation of Δy_t; consequently, good accuracy in the region near the origin may not be expected. This is substantiated by the fact that for the 63-point method the highest effective harmonic would have three waves in the region $0 < x < 0.04$; obviously a sufficient degree of accuracy in the differentiation process cannot be obtained.

As mentioned earlier, Timman's method might be expected to give good results if the size of interval is properly chosen. Inasmuch as only a table for $\Delta\theta = \frac{360}{36} = 10^\circ$ was available, the result of the computation for $\frac{\Delta v}{V_o}$ cannot be expected to be good, as is evidenced by observing figure 7. The result obtained is comparable with that of Multhopp's 15-point and Naiman's 40-point solutions.

An excellent method of examining the accuracy of these methods still further is simply that of solving the inverse problem. From the curves of $\frac{\Delta v}{V_o}$ just discussed, values for $\frac{d(\Delta y_t)}{dx}$ have been computed

and are presented in figure 8. The method of unequal intervals gives good results, indicating that the arrangement of intervals chosen was as good for the inverse problem as for the direct problem. It is apparent that Naiman's method requires even smaller divisions than 160 points in order to avoid inaccuracies near the point $x/c = 0.04$.

The reader may wonder that the inverse problem is not given by Multhopp's method. It must be recalled that Multhopp's method of solving the direct problem does not involve the differentiation of Δy_t; that is, it is particularly fit for this problem and presents, on the other hand, no analogy for the inverse problem:

$$\frac{d(\Delta y_t)}{dx} = \frac{1}{2\pi}\int_0^{2\pi} \frac{\Delta v}{V_o} \cot \frac{\theta - \theta_o}{2}\, d\theta = -\frac{1}{\pi}\int_0^{\pi} \frac{\Delta v}{V_o} \frac{\sin \theta_o}{\cos \theta - \cos \theta_o}\, d\theta$$

Because more extended tables for Timman's method are not available, and the results obtained from the 36-point method for which tables exist are very poor, no further examples of the application of this method will be given.

COMPARISONS OF METHODS OF NAIMAN AND MULTHOPP WITH METHOD EMPLOYING UNEQUAL INTERVALS BASED ON ACTUAL EXAMPLES OF CHANGES IN AIRFOIL SHAPE

The method of unequal intervals has shown good qualities when applied to a problem where the function $\sigma(x)$ is known analytically. However, as mentioned earlier, this function is not usually known in analytic form. This section, therefore, will compare the three principle methods, those of Naiman, Multhopp, and unequal intervals, on the basis of actual design problems, solving the direct problem for $\frac{\Delta v}{V_o}$ and using these results to solve the inverse problem (excluding Multhopp for the inverse problem).

Figure 9(a) shows the Δy_t relations for examples I and II and figure 9(b), the $\frac{d(\Delta y_t)}{dx}$ relations. Note that the slope of $\left[\frac{d(\Delta y_t)}{dx}\right]$ for example II is more than twice that of example I near $x/c = 0$.

The direct problem for example I by Naiman's method is given in figure 10. The 160-point solution does not show any appreciable deviation from the 80-point solutions at the region of $\left(\frac{\Delta v}{V_o}\right)_{max}$; however, near the origin, at $\left(\frac{\Delta v}{V_o}\right)_{min}$, the influence of the smaller-sized intervals ($80:\Delta\theta = 4.5^o$; $160:\Delta\theta = 2.25^o$) is quite pronounced.

The solution by Multhopp's method is given in figure 11;[6] 31 points around the half circle are not sufficient for a solution comparable with Naiman's 80-point solution, and even a solution based on 63 points does not offer much improvement. The results are poor, as might be expected, in the region very near the origin (see preceding section).

Figure 12 presents the results obtained by the method of unequal intervals, compared with results obtained by Naiman's 80- and 160-point solutions. The method of unequal intervals gives results corresponding to those established by Naiman's 160-point solution. The subdivision used is shown in the figure.

As before, the inverse problem was solved, and is given in figure 13. In each case the computed curve of $\frac{\Delta v}{V_o}$ was the one used in obtaining the values for the $\frac{d(\Delta y_t)}{dx}$ curve. Both methods give good results, thus proving that the chosen number of divisions was sufficient in Naiman's method and in the method employing unequal intervals.

The value of $\frac{d(\Delta y_t)}{dx}$ computed at $x/c = 0.171$ is of some interest. This point was computed by the method of unequal intervals in two different ways: First, the arrangement of intervals shown in figure 13 was utilized to compute the lower point. Then a new arrangement of intervals ($\Delta x = 0.018$ for $0 < x < 0.36$) was set up and the same point computed. The idea was to determine the inaccuracies that would result. One might predict that, since the point $x/c = 0.171$ lies at a considerable distance from the region of rapid changes in $\frac{\Delta v}{V_o}$, errors of only small magnitude would be introduced; this is fairly well substantiated by the results shown in the figure because the error thus introduced is approximately that of the deviation of Naiman's 160-point solution.

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[6]Recall that this method does not involve the differentiation of Δy_t.

Now, turning our attention to example II, which, it will be recalled, has a slope of $\frac{d(\Delta y_t)}{dx}$ of approximately twice that of example I, the results given in figures 14 to 17 are obtained.

For the direct problem Naiman's method of 160 points and the method employing unequal intervals give results which are in good agreement. For the inverse problem (fig. 17) it is apparent that the method using unequal intervals is superior $\left(\text{see the deviation at } \left[\frac{d(\Delta y_t)}{dx}\right]_{max} \text{ given by the method of Naiman}\right)$. Multhopp's method gives a rather good result (fig. 15), which may be attained when the Fourier representation of the Δy curve is adequate.

The two examples thus far presented are favorable for Naiman's method because the steep slopes of $\sigma(x)$ occur near $x = 0$ where the points Naiman uses are close together. However, going to still steeper slopes near $x = 0$ would require a rapidly increasing number of points. The new method offers another possibility here. Assume that in that critical region $x_k < x < x_{k+1}$ $(x_k$ may be $0)$ $\sigma(x)$ may be represented by $\sigma(x) = \sum a_n x^n$. Then the integral

$$\int_0^c \frac{\sigma(x)}{x - x_o} dx$$

may be split into three integrals

$$\int_0^c \frac{\sigma(x)}{x - x_o} dx = \int_0^{x_k} \frac{\sigma(x)}{x - x_o} dx + \int_{x_k}^{x_{k+1}} \frac{\sigma(x)}{x - x_o} dx + \int_{x_{k+1}}^c \frac{\sigma(x)}{x - x_o} dx \tag{41}$$

The first and third of these integrals may be solved in the usual manner using the functions j_{no} and j_{no}^*. The second integral will be solved analytically.

This simple form, due to the use of the coordinate x in the Poisson integral, allows a rapid integration, because the integral

$$k_{n,o} = \int_{x_k}^{x_{k+1}} \frac{x^n}{x - x_o}\, dx$$

can be solved by recurrence as follows:

$$k_{n,o} = \frac{{x_{k+1}}^n - {x_k}^n}{n} + x_o k_{n-1,o} \quad \text{for} \quad n \geqq 1 \tag{42}$$

with

$$k_{o,o} = j_{no}\left(\frac{x_k - x_o}{x_{k+1} - x_k}\right) \tag{43}$$

Thus even very steep slopes cause no difficulties.

As already mentioned, examples I and II correspond well to the qualities demanded by Naiman's method insofar as the rather steep slopes occur in those portions where the points θ_n are close together. If those steep slopes should occur in other portions of the chord, however, a very great number of points in the Naiman method would be needed in order to represent $\sigma(x)$ adequately, and to get reliable results. In such a case the method using unequal intervals shows its advantage by allowing a free subdivision of the chord.

A third example will serve to illustrate this. Figure 18 shows a function $\sigma(x) = \frac{d(\Delta y_t)}{dx}$. The essential values of the function lie in a part of the chord where even Naiman's method with 160 points is not sufficient to represent the function accurately. This is forcibly shown by the two curves of $\frac{\Delta v}{V_o}$. If the function $\sigma(x)$ is modified (dotted line) so as to eliminate the high peak, then the $\frac{\Delta v}{V_o}$ curve by unequal intervals can be made to agree with the original $\frac{\Delta v}{V_o}$ by Naiman's 160-point solution, thus definitely proving that, in this example, Naiman's method with 160 points is insufficient.

Table III indicates the computation for the point $x_o = 0.065$ by unequal intervals.

CONCLUDING DISCUSSION

The new method of evaluating the Poisson integral developed herein is to be recommended for all those functions $\sigma(x)$, where steep slopes in small portions of the region to be integrated exist. In these portions a very small size of interval may be chosen without requiring that this same size of interval be used throughout the region of integration. In this manner, the work required for computation may be maintained at a reasonable level even for the most complicated problems.

The analytical treatment of special parts of the integral is possible (evaluating the remainder by the new method; see preceding section). In those problems where a transition to very small intervals in part of the integration range would require the determination of a great many values of σ_n, this idea might be used to advantage.

It should be noted that the smoothness of the function $\sigma(x)$ and its accurate representation by single points is essential for good results. If, for example, single points σ_n are simply taken from a curve for x_n very close to one another it may be compulsory to check these values by a table of differences.

Stanford University
Stanford, Calif., December 6, 1950

APPENDIX A

VALUES OF $b_{\nu n}$ FOR $m_1 = 63$

ν	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	
$b_{\nu\nu}$ / n	326.3974	109.0141	65.8599	47.4876	37.4251	31.1290	26.8610	23.8240	21.5948	19.9200	18.6543	17.6989	16.9936	16.4942	16.1752	16.0192	$b_{\nu\nu}$ / n
2	117.7031	42.5039	2.4288	0.2570	0.1841	0.0810	0.0415	0.0237	0.0146	0.0095	0.0066	0.0048	0.0036	0.0027	0.0021	0.0017	62
4	9.5258	43.5468	26.5068	1.9794	.5176	.2015	.0967	.0530	.0319	.0205	.0140	.0100	.0073	.0056	.0044	.0036	60
6	2.5926	4.3625	26.6882	18.9041	1.6203	.4620	.1920	.0973	.0558	.0347	.0232	.0161	.0118	.0089	.0069	.0055	58
8	1.0658	1.4036	2.8072	19.1646	15.1443	1.3682	.4106	.1786	.0939	.0554	.0356	.0243	.0174	.0129	.0098	.0078	56
10	.5395	.6410	.9490	2.0744	15.1466	12.6165	1.1878	.3703	.1664	.0899	.0544	.0356	.0248	.0181	.0136	.0107	54
12	.3100	.3491	.4522	.7168	1.6503	12.5226	10.8509	1.0599	.3396	.1561	.0862	.0532	.0354	.0251	.0185	.0142	52
14	.1944	.2122	.2553	.3491	.5777	1.3788	10.8939	9.6247	.9643	.3152	.1480	.0831	.0521	.0352	.0253	.0189	50
16	.1300	.1388	.1598	.2011	.2850	.4865	1.1970	9.6917	8.7687	.8896	.2967	.1414	.0806	.0513	.0351	.0255	48
18	.0910	.0960	.1072	.1281	.1663	.2421	.4241	1.0660	8.7421	7.9460	.7702	.2820	.1364	.0788	.0508	.0352	46
20	.0653	.0682	.0748	.0862	.1057	.1406	.2093	.3738	.9538	7.9895	7.4640	.7828	.2685	.1313	.0768	.0501	44
22	.0496	.0514	.0554	.0624	.0736	.0922	.1251	.1896	.3436	.8913	7.5181	7.1773	.7630	.2642	.1308	.0773	42
24	.0380	.0392	.0418	.0462	.0531	.0635	.0816	.1117	.1730	.3182	.8306	7.1818	6.9070	.7392	.2594	.1297	40
26	.0298	.0306	.0324	.0352	.0394	.0464	.0569	.0735	.1029	.1605	.2986	.7948	6.8901	6.6451	.7250	.2566	38
28	.0238	.0243	.0255	.0275	.0305	.0349	.0414	.0514	.0674	.0955	.1505	.2837	.7616	6.6920	6.5771	.7181	36
30	.0190	.0196	.0204	.0218	.0240	.0269	.0313	.0376	.0472	.0628	.0898	.1431	.2722	.7394	6.5335	6.5062	34
32	.0156	.0159	.0168	.0177	.0191	.0212	.0242	.0284	.0346	.0441	.0591	.0855	.1376	.2648	.6216	6.5104	32
34	.0129	.0132	.0136	.0144	.0154	.0170	.0191	.0221	.0262	.0323	.0415	.0564	.0823	.1338	.2597	.7203	30
36	.0108	.0110	.0113	.0119	.0127	.0138	.0153	.0175	.0204	.0245	.0305	.0395	.0541	.0798	.1311	.2572	28
38	.0089	.0091	.0094	.0098	.0105	.0114	.0125	.0140	.0162	.0191	.0230	.0290	.0380	.0526	.0782	.1298	26
40	.0075	.0077	.0079	.0082	.0085	.0093	.0102	.0115	.0130	.0150	.0180	.0219	.0278	.0369	.0515	.0775	24
42	.0064	.0065	.0065	.0069	.0073	.0078	.0085	.0093	.0106	.0121	.0142	.0171	.0211	.0270	.0360	.0509	22
44	.0052	.0053	.0054	.0056	.0059	.0063	.0068	.0075	.0083	.0094	.0110	.0129	.0156	.0194	.0250	.0335	20
46	.0046	.0046	.0047	.0049	.0051	.0054	.0059	.0064	.0071	.0080	.0091	.0107	.0128	.0156	.0198	.0259	18
48	.0038	.0039	.0040	.0041	.0043	.0045	.0049	.0053	.0058	.0065	.0074	.0086	.0102	.0122	.0151	.0193	16
50	.0031	.0031	.0032	.0034	.0036	.0038	.0040	.0043	.0048	.0053	.0060	.0069	.0080	.0096	.0117	.0146	14
52	.0026	.0026	.0026	.0027	.0028	.0030	.0032	.0036	.0039	.0043	.0048	.0055	.0064	.0075	.0090	.0112	12
54	.0020	.0021	.0021	.0022	.0023	.0024	.0026	.0028	.0030	.0034	.0038	.0043	.0050	.0058	.0070	.0085	10
56	.0016	.0016	.0016	.0017	.0018	.0018	.0020	.0021	.0023	.0026	.0028	.0032	.0038	.0044	.0052	.0063	8
58	.0012	.0012	.0012	.0012	.0013	.0014	.0014	.0016	.0017	.0018	.0021	.0023	.0027	.0031	.0038	.0045	6
60	.0008	.0008	.0008	.0008	.0008	.0009	.0009	.0010	.0011	.0012	.0013	.0015	.0017	.0020	.0024	.0028	4
62	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005	.0006	.0007	.0007	.0008	.0010	.0012	.0014	2
	63	61	59	57	55	53	51	49	47	45	43	41	39	37	35	33	ν

ν	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	
n \ $b_{\nu\nu}$	163.3820	82.0134	55.1021	41.8104	33.9472	28.7992	25.2183	22.6273	20.6986	19.4730	18.1414	17.3183	16.7205	16.3135	16.0773	16.0000	$b_{\nu\nu}$ / n
1	58.9176	2.3935	0.4377	0.1365	0.0561	0.0274	0.0150	0.0090	0.0058	0.0039	0.0028	0.0020	0.0015	0.0012	0.0009	0.0008	63
3	63.7000	32.7600	2.2050	.5383	.1996	.0922	.0491	.0288	.0182	.0122	.0086	.0062	.0047	.0036	.0029	.0023	61
5	6.0252	31.6325	22.3288	1.7821	.4892	.1977	.0978	.0549	.0337	.0221	.0153	.0110	.0082	.0063	.0050	.0040	59
7	1.8280	3.4185	21.9354	16.9822	1.4830	.4347	.1854	.0958	.0558	.0354	.0238	.0168	.0124	.0094	.0074	.0059	57
9	.8038	1.1341	2.3857	17.1282	13.6326	1.2700	.3893	.1723	.0919	.0550	.0357	.0246	.0177	.0133	.0103	.0082	55
11	.4254	.5312	.8178	1.8378	13.8466	11.6391	1.1170	.3536	.1610	.0879	.0538	.0356	.0249	.0183	.0139	.0109	53
13	.2526	.2953	.3940	.6392	1.5011	11.6339	10.2276	1.0073	.3268	.1517	.0845	.0526	.0354	.0252	.0187	.0144	51
15	.1623	.1824	.2250	.3135	.5276	1.2798	10.1883	9.2053	.9262	.3055	.1444	.0818	.0517	.0352	.0254	.0191	49
17	.1106	.1211	.1422	.1819	.2620	.4529	1.1262	9.1881	8.3891	.8601	.2886	.1387	.0764	.0510	.0352	.0257	47
19	.0786	.0846	.0962	.1165	.1531	.2257	.3990	1.0105	8.3667	7.7950	.8118	.2767	.1346	.0782	.0506	.0354	45
21	.0579	.0615	.0683	.0838	.0990	.1331	.1999	.3598	.9249	7.7919	7.3235	.7760	.2677	.1317	.0774	.0507	43
23	.0438	.0461	.0504	.0573	.0684	.0866	.1183	.1807	.3299	.8613	7.3568	7.0274	.7509	.2615	.1300	.0773	41
25	.0340	.0355	.0383	.0427	.0495	.0602	.0774	.1074	.1662	.3078	.8146	7.0388	6.7793	.7312	.2575	.1296	39
27	.0268	.0278	.0297	.0327	.0371	.0438	.0539	.0704	.0989	.1551	.2906	.7761	6.7364	6.6188	.7207	.2568	37
29	.0215	.0222	.0235	.0256	.0286	.0330	.0394	.0492	.0650	.0925	.1467	.2777	.7496	6.6335	6.4941	.7175	35
31	.0174	.0180	.0189	.0204	.0225	.0255	.0298	.0360	.0455	.0608	.0875	.1402	.2679	.7313	6.5297	6.5025	33
33	.0143	.0147	.0154	.0165	.0180	.0201	.0231	.0273	.0334	.0427	.0576	.0837	.1355	.2619	.7228	6.5025	31
35	.0118	.0122	.0127	.0135	.0146	.0162	.0183	.0212	.0253	.0313	.0404	.0551	.0809	.1322	.2581	.6148	29
37	.0099	.0101	.0105	.0111	.0120	.0131	.0147	.0168	.0197	.0238	.0297	.0387	.0533	.0790	.1304	.7568	27
39	.0083	.0085	.0088	.0092	.0099	.0108	.0119	.0135	.0156	.0185	.0225	.0284	.0374	.0520	.0778	.1296	25
41	.0070	.0071	.0074	.0077	.0082	.0089	.0098	.0110	.0125	.0146	.0175	.0215	.0274	.0364	.0512	.0773	23
43	.0059	.0060	.0062	.0065	.0069	.0074	.0081	.0090	.0102	.0117	.0138	.0167	.0207	.0266	.0358	.0507	21
45	.0050	.0050	.0052	.0054	.0057	.0062	.0067	.0074	.0083	.0095	.0110	.0131	.0160	.0201	.0261	.0354	19
47	.0042	.0042	.0044	.0045	.0048	.0051	.0055	.0061	.0068	.0077	.0089	.0104	.0125	.0154	.0196	.0257	17
49	.0035	.0035	.0036	.0038	.0040	.0042	.0046	.0050	.0055	.0062	.0071	.0083	.0099	.0120	.0149	.0191	15
51	.0029	.0029	.0030	.0031	.0032	.0035	.0037	.0041	.0045	.0050	.0057	.0066	.0078	.0093	.0115	.0144	13
53	.0023	.0024	.0024	.0025	.0026	.0028	.0030	.0033	.0036	.0040	.0045	.0052	.0061	.0072	.0088	.0109	11
55	.0018	.0019	.0019	.0020	.0021	.0022	.0024	.0026	.0028	.0031	.0035	.0040	.0047	.0055	.0066	.0082	9
57	.0014	.0014	.0015	.0015	.0016	.0017	.0018	.0019	.0021	.0023	.0026	.0030	.0035	.0041	.0049	.0059	7
59	.0010	.0010	.0010	.0010	.0011	.0012	.0012	.0013	.0015	.0016	.0018	.0021	.0024	.0028	.0033	.0040	5
61	.0006	.0006	.0006	.0006	.0006	.0007	.0007	.0008	.0009	.0010	.0011	.0012	.0014	.0016	.0019	.0023	3
63	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0003	.0003	.0003	.0004	.0004	.0005	.0005	.0006	.0008	1
	62	60	58	56	54	52	50	48	46	44	42	40	38	36	34	32	ν

APPENDIX B

VALUES OF j_{no} AND j_{no}^* AS FUNCTIONS OF $\frac{x_n - x_o}{\Delta x}$

The values of the functions j_{no} and j_{no}^* are presented as indicated in the following table. The values are tabulated in a form selected to minimize the necessity for interpolation except for the region containing the singularities of the functions j_{no} and j_{no}^*. For ease in computation, tables B-I to B-VIII, inclusive, are arranged so that the vertical increment of $\frac{x_n - x_o}{\Delta x}$ is unity. Table B-IX gives additional values for the region containing the singularities of the functions j_{no} and j_{no}^*.

TABLE NO.	RANGE OF $\frac{x_n - x_o}{\Delta x}$	INCREMENT OF $\frac{x_n - x_o}{\Delta x}$
B-I	-189 to -90	1.0
B-II	-89.5 to -40.0	.5
B-III	-39.9 to -20.0	.1
B-IV	-19.99 to 0	.01
B-V	0 to 19.99	.01
B-VI	20.0 to 39.9	.1
B-VII	40.0 to 89.5	.5
B-VIII	90 to 189	1.0
B-IX	-1.000 to 0.000	0.001

TABLE B-I.- VALUES OF j_{no} AND j_{no}^* USED IN EVALUATING EQUATION (26)

$$-189 \leqq \frac{x_n - x_o}{\Delta x} \leqq -90$$

$\frac{x_n-x_o}{\Delta x}$ \ X	-18X.0		-1		-16X.0		-15X.0		-14X.0		-13X.0		-12X.0		-11X.0		-10X.0		-9X.0	
	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
9	-0.0053	-0.0027	-0.0056	-0.0028	-0.0059	-0.0030	-0.0063	-0.0032	-0.0067	-0.0034	-0.0072	-0.0036	-0.0078	-0.0039	-0.0084	-0.0042	-0.0092	-0.0046	-0.0102	-0.0051
8	-.0053	-.0027	-.0056	-.0028	-.0060	-.0030	-.0063	-.0032	-.0068	-.0034	-.0073	-.0036	-.0078	-.0039	-.0085	-.0043	-.0093	-.0047	-.0103	-.0051
7	-.0054	-.0027	-.0057	-.0028	-.0060	-.0030	-.0064	-.0032	-.0068	-.0034	-.0073	-.0037	-.0079	-.0040	-.0086	-.0043	-.0094	-.0047	-.0104	-.0052
6	-.0054	-.0027	-.0057	-.0029	-.0060	-.0030	-.0064	-.0032	-.0069	-.0034	-.0074	-.0037	-.0080	-.0040	-.0087	-.0043	-.0095	-.0047	-.0105	-.0053
5	-.0054	-.0027	-.0057	-.0029	-.0061	-.0030	-.0065	-.0032	-.0069	-.0035	-.0074	-.0037	-.0080	-.0040	-.0087	-.0044	-.0096	-.0047	-.0106	-.0053
4	-.0054	-.0027	-.0058	-.0029	-.0061	-.0031	-.0065	-.0033	-.0070	-.0035	-.0075	-.0037	-.0081	-.0041	-.0088	-.0044	-.0097	-.0048	-.0107	-.0053
3	-.0055	-.0027	-.0058	-.0029	-.0062	-.0031	-.0066	-.0033	-.0070	-.0035	-.0075	-.0038	-.0082	-.0041	-.0089	-.0045	-.0098	-.0049	-.0108	-.0054
2	-.0055	-.0028	-.0058	-.0029	-.0062	-.0031	-.0066	-.0033	-.0071	-.0035	-.0076	-.0038	-.0082	-.0041	-.0090	-.0045	-.0099	-.0049	-.0109	-.0055
1	-.0055	-.0028	-.0059	-.0029	-.0062	-.0031	-.0066	-.0033	-.0071	-.0036	-.0077	-.0038	-.0083	-.0042	-.0090	-.0045	-.0100	-.0050	-.0110	-.0055
0	-.0056	-.0028	-.0059	-.0030	-.0063	-.0031	-.0067	-.0033	-.0072	-.0036	-.0077	-.0039	-.0084	-.0042	-.0091	-.0046	-.0101	-.0050	-.0112	-.0056

TABLE B-II.- VALUES OF j_{no} AND j_{no}^* USED IN EVALUATING EQUATION (26)

$$-89.5 \leqq \frac{x_n - x_o}{\Delta x} \leqq -40.0$$

$\frac{x_n-x_o}{\Delta x}$ \ X	-8X.5		-8X.0		-7X.5		-7X.0		-6X.5		-6X.0		-5X.5		-5X.0		-4X.5		-4X.0	
	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
9	-0.0112	-0.0056	-0.0113	-0.0057	-0.0127	-0.0064	-0.0127	-0.0064	-0.0145	-0.0073	-0.0146	-0.0073	-0.0169	-0.0085	-0.0171	-0.0085	-0.0204	-0.0102	-0.0206	-0.0103
8	-.0114	-.0057	-.0114	-.0058	-.0128	-.0064	-.0129	-.0064	-.0147	-.0074	-.0148	-.0074	-.0172	-.0086	-.0174	-.0087	-.0208	-.0104	-.0211	-.0105
7	-.0115	-.0058	-.0116	-.0058	-.0130	-.0065	-.0131	-.0065	-.0149	-.0075	-.0150	-.0075	-.0175	-.0088	-.0177	-.0089	-.0213	-.0107	-.0215	-.0108
6	-.0116	-.0058	-.0117	-.0059	-.0132	-.0066	-.0132	-.0067	-.0152	-.0076	-.0153	-.0076	-.0179	-.0090	-.0180	-.0090	-.0217	-.0109	-.0220	-.0110
5	-.0118	-.0059	-.0118	-.0059	-.0133	-.0067	-.0134	-.0067	-.0154	-.0077	-.0155	-.0078	-.0182	-.0092	-.0184	-.0093	-.0222	-.0112	-.0225	-.0112
4	-.0119	-.0059	-.0120	-.0060	-.0135	-.0067	-.0136	-.0068	-.0156	-.0078	-.0157	-.0079	-.0185	-.0093	-.0187	-.0094	-.0227	-.0114	-.0230	-.0116
3	-.0120	-.0060	-.0121	-.0061	-.0137	-.0068	-.0138	-.0069	-.0159	-.0079	-.0160	-.0080	-.0189	-.0094	-.0190	-.0095	-.0233	-.0117	-.0235	-.0118
2	-.0122	-.0061	-.0123	-.0061	-.0139	-.0070	-.0140	-.0070	-.0161	-.0081	-.0163	-.0081	-.0192	-.0096	-.0194	-.0097	-.0238	-.0120	-.0241	-.0121
1	-.0123	-.0062	-.0124	-.0062	-.0141	-.0071	-.0142	-.0071	-.0164	-.0082	-.0165	-.0083	-.0196	-.0099	-.0198	-.0100	-.0244	-.0122	-.0247	-.0124
0	-.0125	-.0062	-.0126	-.0063	-.0143	-.0071	-.0144	-.0072	-.0167	-.0084	-.0168	-.0084	-.0200	-.0101	-.0202	-.0102	-.0250	-.0125	-.0253	-.0127

TABLE B-III.- VALUES OF j_{no} AND $j_{no}{}^*$ USED IN EVALUATING EQUATION (26)

$$-39.9 \leqq \frac{x_n - x_o}{\Delta x} \leqq -20.0$$

X	9		8		7		6		5		4		3		2		1		0	
$\frac{x_n-x_o}{\Delta x}$	j_{no}	$j_{no}{}^*$	j_{no}	$j_{no}{}^*$	j_{no}	$j_{no}{}^*$	j_{no}	$j_{no}{}^*$	j_{no}	$j_{no}{}^*$	j_{no}	$j_{no}{}^*$	j_{no}	$j_{no}{}^*$	j_{no}	$j_{no}{}^*$	j_{no}	$j_{no}{}^*$	j_{no}	$j_{no}{}^*$
-39.X	-0.0254	-0.0127	-0.0254	-0.0128	-0.0255	-0.0128	-0.0256	-0.0128	-0.0256	-0.0129	-0.0257	-0.0129	-0.0258	-0.0130	-0.0258	-0.0130	-0.0259	-0.0130	-0.0260	-0.0131
-38.	-.0260	-.0131	-.0261	-.0131	-.0262	-.0132	-.0262	-.0132	-.0263	-.0132	-.0264	-.0133	-.0265	-.0133	-.0265	-.0133	-.0266	-.0134	-.0267	-.0134
-37.	-.0267	-.0134	-.0268	-.0135	-.0269	-.0135	-.0270	-.0135	-.0270	-.0136	-.0271	-.0137	-.0272	-.0137	-.0273	-.0137	-.0273	-.0137	-.0274	-.0138
-36.	-.0275	-.0138	-.0276	-.0138	-.0276	-.0139	-.0277	-.0139	-.0278	-.0139	-.0279	-.0140	-.0279	-.0140	-.0280	-.0141	-.0281	-.0141	-.0282	-.0141
-35.	-.0283	-.0142	-.0283	-.0142	-.0284	-.0142	-.0285	-.0143	-.0286	-.0143	-.0287	-.0144	-.0287	-.0145	-.0288	-.0145	-.0289	-.0145	-.0290	-.0146
-34.	-.0291	-.0146	-.0292	-.0146	-.0292	-.0147	-.0293	-.0147	-.0294	-.0148	-.0295	-.0148	-.0296	-.0149	-.0297	-.0149	-.0298	-.0150	-.0299	-.0150
-33.	-.0299	-.0150	-.0300	-.0151	-.0301	-.0151	-.0302	-.0152	-.0303	-.0153	-.0304	-.0153	-.0305	-.0154	-.0306	-.0154	-.0307	-.0154	-.0308	-.0155
-32.	-.0309	-.0156	-.0310	-.0156	-.0311	-.0156	-.0312	-.0157	-.0313	-.0157	-.0314	-.0158	-.0315	-.0158	-.0315	-.0158	-.0316	-.0159	-.0317	-.0160
-31.	-.0319	-.0160	-.0320	-.0161	-.0321	-.0161	-.0322	-.0162	-.0323	-.0162	-.0324	-.0163	-.0325	-.0163	-.0326	-.0164	-.0327	-.0164	-.0328	-.0165
-30.	-.0329	-.0165	-.0330	-.0166	-.0331	-.0166	-.0332	-.0167	-.0333	-.0167	-.0334	-.0168	-.0336	-.0169	-.0337	-.0169	-.0338	-.0170	-.0339	-.0171
-29.	-.0340	-.0171	-.0341	-.0172	-.0342	-.0172	-.0344	-.0173	-.0345	-.0173	-.0346	-.0174	-.0347	-.0175	-.0348	-.0175	-.0350	-.0176	-.0351	-.0176
-28.	-.0352	-.0177	-.0353	-.0178	-.0355	-.0178	-.0356	-.0179	-.0357	-.0180	-.0358	-.0180	-.0360	-.0181	-.0361	-.0182	-.0362	-.0182	-.0364	-.0183
-27.	-.0365	-.0184	-.0366	-.0184	-.0368	-.0185	-.0369	-.0186	-.0370	-.0186	-.0372	-.0187	-.0373	-.0188	-.0375	-.0188	-.0376	-.0189	-.0377	-.0190
-26.	-.0379	-.0191	-.0380	-.0191	-.0382	-.0192	-.0383	-.0193	-.0385	-.0193	-.0386	-.0194	-.0388	-.0195	-.0389	-.0196	-.0391	-.0196	-.0392	-.0197
-25.	-.0394	-.0198	-.0395	-.0199	-.0397	-.0200	-.0398	-.0200	-.0400	-.0201	-.0402	-.0202	-.0403	-.0203	-.0405	-.0204	-.0407	-.0205	-.0408	-.0206
-24.	-.0410	-.0206	-.0412	-.0207	-.0413	-.0208	-.0415	-.0209	-.0417	-.0210	-.0418	-.0211	-.0420	-.0211	-.0422	-.0212	-.0424	-.0213	-.0426	-.0214
-23.	-.0427	-.0215	-.0429	-.0216	-.0431	-.0217	-.0433	-.0218	-.0435	-.0219	-.0437	-.0219	-.0439	-.0221	-.0441	-.0222	-.0443	-.0223	-.0444	-.0223
-22.	-.0447	-.0225	-.0449	-.0226	-.0451	-.0227	-.0453	-.0228	-.0455	-.0229	-.0457	-.0230	-.0459	-.0231	-.0461	-.0232	-.0463	-.0233	-.0465	-.0235
-21.	-.0467	-.0236	-.0470	-.0237	-.0472	-.0238	-.0474	-.0239	-.0476	-.0240	-.0479	-.0241	-.0481	-.0242	-.0483	-.0243	-.0486	-.0244	-.0488	-.0246
-20.X	-.0490	-.0247	-.0493	-.0248	-.0495	-.0250	-.0498	-.0251	-.0500	-.0252	-.0503	-.0253	-.0505	-.0255	-.0508	-.0256	-.0510	-.0257	-.0513	-.0259

TABLE B-IV.– VALUES OF j_{no} AND j_{no}^* USED IN EVALUATING EQUATION (26)

(a) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $99 \geqq XX \geqq 90$

XX	99		98		97		96		95		94		93		92		91		90	
$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
–19.XX	–0.0513	–0.0259	–0.0513	–0.0259	–0.0514	–0.0259	–0.0514	–0.0259	–0.0514	–0.0259	–0.0515	–0.0259	–0.0515	–0.0260	–0.0515	–0.0260	–0.0515	–0.0260	–0.0516	–0.0260
–18.	–.0541	–.0273	–.0541	–.0273	–.0542	–.0273	–.0542	–.0273	–.0542	–.0273	–.0542	–.0274	–.0543	–.0274	–.0543	–.0274	–.0543	–.0274	–.0544	–.0274
–17.	–.0572	–.0289	–.0572	–.0289	–.0573	–.0289	–.0573	–.0289	–.0573	–.0289	–.0574	–.0289	–.0574	–.0290	–.0574	–.0290	–.0575	–.0290	–.0575	–.0290
–16.	–.0607	–.0306	–.0607	–.0307	–.0607	–.0307	–.0608	–.0307	–.0608	–.0307	–.0608	–.0307	–.0609	–.0307	–.0609	–.0308	–.0610	–.0308	–.0610	–.0308
–15.	–.0646	–.0326	–.0646	–.0327	–.0647	–.0327	–.0647	–.0327	–.0647	–.0327	–.0648	–.0327	–.0648	–.0328	–.0649	–.0328	–.0649	–.0328	–.0650	–.0328
–14.	–.0690	–.0349	–.0691	–.0349	–.0691	–.0350	–.0692	–.0350	–.0692	–.0350	–.0693	–.0350	–.0693	–.0351	–.0694	–.0351	–.0694	–.0351	–.0695	–.0351
–13.	–.0742	–.0375	–.0742	–.0376	–.0743	–.0376	–.0743	–.0376	–.0744	–.0376	–.0744	–.0377	–.0745	–.0377	–.0746	–.0377	–.0746	–.0378	–.0747	–.0378
–12.	–.0801	–.0406	–.0802	–.0406	–.0802	–.0406	–.0803	–.0407	–.0804	–.0407	–.0804	–.0407	–.0805	–.0408	–.0806	–.0408	–.0806	–.0408	–.0807	–.0408
–11.	–.0871	–.0441	–.0872	–.0442	–.0872	–.0442	–.0873	–.0443	–.0874	–.0443	–.0875	–.0443	–.0875	–.0444	–.0876	–.0444	–.0877	–.0445	–.0878	–.0445
–10.	–.0954	–.0485	–.0955	–.0485	–.0956	–.0485	–.0957	–.0486	–.0958	–.0486	–.0959	–.0486	–.0959	–.0487	–.0960	–.0488	–.0961	–.0489	–.0962	–.0489
–9.	–.1055	–.0536	–.1057	–.0538	–.1057	–.0538	–.1058	–.0539	–.1059	–.0539	–.1060	–.0540	–.1061	–.0540	–.1063	–.0541	–.1064	–.0541	–.1065	–.0542
–8.	–.1179	–.0601	–.1181	–.0602	–.1182	–.0603	–.1183	–.0604	–.1185	–.0604	–.1186	–.0605	–.1188	–.0605	–.1189	–.0606	–.1190	–.0607	–.1192	–.0608
–7.	–.1337	–.0683	–.1339	–.0684	–.1341	–.0685	–.1343	–.0686	–.1344	–.0687	–.1346	–.0688	–.1348	–.0689	–.1350	–.0690	–.1352	–.0691	–.1353	–.0692
–6.	–.1544	–.0792	–.1546	–.0793	–.1549	–.0794	–.1551	–.0796	–.1554	–.0797	–.1556	–.0798	–.1558	–.0799	–.1561	–.0801	–.1563	–.0802	–.1565	–.0803
–5.	–.1827	–.0941	–.1830	–.0943	–.1833	–.0945	–.1837	–.0946	–.1840	–.0948	–.1843	–.0950	–.1847	–.0952	–.1850	–.0954	–.1854	–.0955	–.1857	–.0957
–4.	–.2236	–.1160	–.2242	–.1163	–.2247	–.1165	–.2252	–.1168	–.2257	–.1171	–.2262	–.1174	–.2267	–.1176	–.2272	–.1179	–.2277	–.1182	–.2283	–.1185
–3.	–.2885	–.1512	–.2894	–.1516	–.2902	–.1521	–.2911	–.1526	–.2919	–.1530	–.2928	–.1535	–.2936	–.1540	–.2945	–.1545	–.2954	–.1549	–.2963	–.1554
–2.	–.4071	–.2173	–.4088	–.2183	–.4105	–.2193	–.4122	–.2202	–.4140	–.2212	–.4157	–.2222	–.4175	–.2232	–.4193	–.2242	–.4211	–.2253	–.4229	–.2263
–1.	–.6982	–.3894	–.7033	–.3925	–.7085	–.3957	–.7138	–.3990	–.7191	–.4023	–.7245	–.4056	–.7301	–.4091	–.7357	–.4126	–.7414	–.4161	–.7472	–.4197
–0.XX	–4.5951	–3.5492	–3.8918	–2.8140	–3.4761	–2.3718	–3.1781	–2.0509	–2.9444	–1.7972	–2.7516	–1.5865	–2.5867	–1.4056	–2.4423	–1.2470	–2.3136	–1.1054	–2.1972	–.9775

TABLE B-IV.– CONTINUED

(b) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $89 \geqq XX \geqq 80$

XX	89		88		87		86		85		84		83		82		81		80	
$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
–19.XX	–0.0516	–0.0260	–0.0516	–0.0260	–0.0516	–0.0260	–0.0517	–0.0260	–0.0517	–0.0261	–0.0517	–0.0261	–0.0517	–0.0261	–0.0518	–0.0261	–0.0518	–0.0261	–0.0518	–0.0261
–18.	–.0544	–.0274	–.0544	–.0275	–.0545	–.0275	–.0545	–.0275	–.0545	–.0275	–.0545	–.0275	–.0546	–.0275	–.0546	–.0276	–.0546	–.0276	–.0547	–.0276
–17.	–.0575	–.0290	–.0576	–.0291	–.0576	–.0291	–.0576	–.0291	–.0577	–.0291	–.0577	–.0291	–.0577	–.0291	–.0578	–.0292	–.0578	–.0292	–.0578	–.0292
–16.	–.0610	–.0308	–.0611	–.0308	–.0611	–.0309	–.0611	–.0309	–.0612	–.0309	–.0612	–.0309	–.0613	–.0310	–.0613	–.0310	–.0613	–.0310	–.0614	–.0310
–15.	–.0650	–.0328	–.0650	–.0329	–.0651	–.0329	–.0651	–.0329	–.0652	–.0329	–.0652	–.0330	–.0653	–.0330	–.0653	–.0330	–.0653	–.0330	–.0654	–.0331
–14.	–.0695	–.0352	–.0696	–.0352	–.0696	–.0352	–.0697	–.0352	–.0697	–.0353	–.0698	–.0353	–.0698	–.0353	–.0699	–.0353	–.0699	–.0354	–.0700	–.0354
–13.	–.0747	–.0378	–.0748	–.0378	–.0748	–.0379	–.0749	–.0379	–.0749	–.0379	–.0750	–.0380	–.0751	–.0380	–.0751	–.0380	–.0752	–.0380	–.0752	–.0381
–12.	–.0808	–.0409	–.0808	–.0410	–.0809	–.0410	–.0810	–.0411	–.0810	–.0411	–.0811	–.0411	–.0811	–.0411	–.0812	–.0411	–.0813	–.0412	–.0814	–.0413
–11.	–.0879	–.0446	–.0879	–.0447	–.0880	–.0447	–.0881	–.0447	–.0882	–.0447	–.0882	–.0448	–.0883	–.0448	–.0884	–.0448	–.0885	–.0449	–.0886	–.0449
–10.	–.0963	–.0489	–.0964	–.0489	–.0965	–.0491	–.0966	–.0491	–.0967	–.0491	–.0968	–.0492	–.0969	–.0492	–.0970	–.0492	–.0971	–.0493	–.0972	–.0494
–9.	–.1066	–.0543	–.1067	–.0543	–.1068	–.0544	–.1069	–.0545	–.1071	–.0545	–.1072	–.0545	–.1073	–.0546	–.1074	–.0547	–.1075	–.0548	–.1076	–.0548
–8.	–.1193	–.0608	–.1195	–.0609	–.1196	–.0610	–.1198	–.0611	–.1199	–.0612	–.1200	–.0612	–.1202	–.0613	–.1203	–.0614	–.1205	–.0614	–.1206	–.0616
–7.	–.1355	–.0693	–.1357	–.0694	–.1359	–.0695	–.1361	–.0696	–.1363	–.0697	–.1365	–.0698	–.1366	–.0699	–.1368	–.0700	–.1370	–.0701	–.1372	–.0702
–6.	–.1568	–.0805	–.1571	.0806	–.1573	–.0807	–.1576	–.0809	–.1578	–.0810	–.1581	–.0811	–.1583	–.0813	–.1586	–.0814	–.1588	–.0815	–.1591	–.0816
–5.	–.1861	–.0959	–.1864	–.0961	–.1868	–.0963	–.1871	–.0965	–.1875	–.0967	–.1878	–.0968	–.1882	–.0970	–.1885	–.0972	–.1889	–.0974	–.1892	–.0976
–4.	–.2288	–.1187	–.2293	–.1190	–.2298	–.1193	–.2304	–.1196	–.2309	–.1199	–.2314	–.1202	–.2320	–.1205	–.2325	–.1208	–.2331	–.1211	–.2336	–.1213
–3.	–.2972	–.1559	–.2980	–.1564	–.2989	–.1569	–.2998	–.1574	–.3008	–.1579	–.3017	–.1584	–.3026	–.1589	–.3035	–.1594	–.3044	–.1599	–.3054	–.1604
–2.	–.4247	–.2273	–.4265	–.2284	–.4284	–.2294	–.4302	–.2305	–.4321	–.2316	–.4340	–.2327	–.4360	–.2338	–.4379	–.2349	–.4399	–.2360	–.4418	–.2371
–1.	–.7531	–.4234	–.7591	–.4271	–.7652	–.4309	–.7714	–.4348	–.7777	–.4388	–.7841	–.4428	–.7906	–.4469	–.7973	–.4511	–.8041	–.4554	–.8109	–.4597
–0.XX	–2.0908	–.8608	–1.9924	–.7533	–1.9010	–.6538	–1.8153	–.5612	–1.7346	–.4744	–1.6582	–.3929	–1.5856	–.3161	–1.5164	–.2434	–1.4500	–.1745	–1.3863	–.1090

TABLE B-IV.— CONTINUED

(c) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $79 \geqq XX \geqq 70$

XX / $\frac{x_n-x_o}{\Delta x}$	79		78		77		76		75		74		73		72		71		70	
	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
−19.XX	−0.0519	−0.0262	−0.0519	−0.0262	−0.0519	−0.0262	−0.0519	−0.0262	−0.0520	−0.0262	−0.0520	−0.0262	−0.0520	−0.0262	−0.0520	−0.0262	−0.0521	−0.0263	−0.0521	−0.0263
−18.	−.0547	−.0276	−.0547	−.0276	−.0547	−.0276	−.0548	−.0276	−.0548	−.0277	−.0548	−.0277	−.0549	−.0277	−.0549	−.0277	−.0549	−0.0277	−.0550	−.0277
−17.	−.0579	−.0292	−.0579	−.0292	−.0579	−.0292	−.0580	−.0293	−.0580	−.0293	−.0580	−.0293	−.0581	−.0293	−.0581	−.0293	−.0581	−.0293	−.0582	−.0294
−16.	−.0614	−.0310	−.0614	−.0310	−.0615	−.0311	−.0615	−.0311	−.0616	−.0311	−.0616	−.0311	−.0616	−.0311	−.0617	−.0312	−.0617	−.0312	−.0617	−.0312
−15.	−.0654	−.0331	−.0655	−.0331	−.0655	−.0331	−.0656	−.0331	−.0656	−.0332	−.0656	−.0332	−.0657	−.0332	−.0657	−.0332	−.0658	−.0332	−.0658	−.0333
−14.	−.0700	−.0354	−.0701	−.0354	−.0701	−.0355	−.0702	−.0355	−.0702	−.0355	−.0703	−.0355	−.0703	−.0356	−.0704	−.0356	−.0704	−.0356	−.0705	−.0356
−13.	−.0753	−.0381	−.0753	−.0381	−.0754	−.0382	−.0755	−.0382	−.0755	−.0382	−.0756	−.0383	−.0756	−.0383	−.0757	−.0383	−.0757	−.0383	−.0758	−.0384
−12.	−.0814	−.0413	−.0815	−.0413	−.0815	−.0413	−.0816	−.0413	−.0817	−.0414	−.0817	−.0414	−.0818	−.0414	−.0819	−.0415	−.0819	−.0415	−.0820	−.0416
−11.	−.0886	−.0449	−.0887	−.0450	−.0888	−.0451	−.0889	−.0451	−.0889	−.0451	−.0890	−.0451	−.0891	−.0452	−.0892	−.0452	−.0893	−.0453	−.0893	−.0454
−10.	−.0973	−.0495	−.0974	−.0495	−.0974	−.0495	−.0975	−.0496	−.0976	−.0496	−.0977	−.0497	−.0978	−.0497	−.0979	−.0498	−.0980	−.0498	−.0981	−.0498
−9.	−.1078	−.0549	−.1079	−.0549	−.1080	−.0550	−.1081	−.0551	−.1082	−.0551	−.1083	−.0552	−.1085	−.0552	−.1086	−.0553	−.1087	−.0553	−.1088	−.0554
−8.	−.1208	−.0616	−.1209	−.0616	−.1211	−.0618	−.1212	−.0619	−.1214	−.0619	−.1215	−.0620	−.1217	−.0621	−.1218	−.0621	−.1220	−.0622	−.1221	−.0623
−7.	−.1374	−.0703	−.1376	−.0704	−.1378	−.0705	−.1380	−.0706	−.1382	−.0707	−.1383	−.0708	−.1385	−.0709	−.1387	−.0710	−.1389	−.0711	−.1391	−.0711
−6.	−.1593	−.0818	−.1596	−.0819	−.1598	−.0820	−.1601	−.0822	−.1603	−.0823	−.1606	−.0825	−.1609	−.0826	−.1611	−.0828	−.1614	−.0829	−.1616	−.0830
−5.	−.1896	−.0978	−.1900	−.0980	−.1903	−.0982	−.1907	−.0984	−.1911	−.0986	−.1914	−.0988	−.1918	−.0989	−.1922	−.0991	−.1925	−.0994	−.1929	−.0996
−4.	−.2342	−.1216	−.2347	−.1219	−.2353	−.1222	−.2358	−.1226	−.2364	−.1229	−.2370	−.1232	−.2375	−.1235	−.2381	−.1238	−.2387	−.1241	−.2392	−.1244
−3.	−.3063	−.1610	−.3073	−.1615	−.3082	−.1620	−.3092	−.1626	−.3102	−.1631	−.3111	−.1636	−.3121	−.1642	−.3131	−.1647	−.3141	−.1652	−.3151	−.1658
−2.	−.4438	−.2383	−.4458	−.2394	−.4479	−.2406	−.4499	−.2418	−.4520	−.2430	−.4541	−.2442	−.4562	−.2454	−.4583	−.2466	−.4605	−.2478	−.4626	−.2491
−1.	−.8179	−.4641	−.8251	−.4687	−.8323	−.4732	−.8398	−.4780	−.8473	−.4827	−.8550	−.4877	−.8628	−.4927	−.8708	−.4978	−.8790	−.5031	−.8873	−.5084
−0.XX	−1.3249	−.0467	−1.2657	.0128	−1.2083	.0696	−1.1527	.1240	−1.0986	.1760	−1.0460	.2260	−.9946	.2739	−.9445	.3200	−.8954	.3643	−.8473	.4069

TABLE B-IV.— CONTINUED

(d) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $69 \geqq XX \geqq 60$

XX / $\frac{x_n-x_o}{\Delta x}$	69		68		67		66		65		64		63		62		61		60	
	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
−19.XX	−0.0521	−0.0263	−0.0521	−0.0263	−0.0522	−0.0263	−0.0522	−0.0263	−0.0522	−0.0263	−0.0523	−0.0263	−0.0523	−0.0264	−0.0523	−0.0264	−0.0523	−0.0264	−0.0524	−0.0264
−18.	−.0550	−.0277	−.0550	−.0278	−.0550	−.0278	−.0551	−.0278	−.0551	−.0278	−.0551	−.0278	−.0552	−.0278	−.0552	−.0279	−.0552	−.0279	−.0553	−.0279
−17.	−.0582	−.0294	−.0582	−.0294	−.0583	−.0294	−.0583	−.0294	−.0583	−.0294	−.0584	−.0295	−.0584	−.0295	−.0584	−.0295	−.0585	−.0295	−.0585	−.0295
−16.	−.0618	−.0312	−.0618	−.0312	−.0619	−.0313	−.0619	−.0313	−.0619	−.0313	−.0620	−.0313	−.0620	−.0313	−.0621	−.0313	−.0621	−.0314	−.0621	−.0314
−15.	−.0659	−.0333	−.0659	−.0333	−.0659	−.0333	−.0660	−.0334	−.0660	−.0334	−.0661	−.0334	−.0661	−.0334	−.0662	−.0334	−.0662	−.0335	−.0662	−.0335
−14.	−.0705	−.0357	−.0706	−.0357	−.0706	−.0357	−.0707	−.0357	−.0707	−.0358	−.0708	−.0358	−.0708	−.0358	−.0709	−.0358	−.0709	−.0359	−.0710	−.0359
−13.	−.0758	−.0384	−.0759	−.0384	−.0760	−.0385	−.0760	−.0385	−.0761	−.0385	−.0761	−.0386	−.0762	−.0386	−.0763	−.0386	−.0763	−.0386	−.0764	−.0387
−12.	−.0821	−.0416	−.0822	−.0417	−.0822	−.0417	−.0823	−.0417	−.0823	−.0417	−.0824	−.0418	−.0825	−.0418	−.0826	−.0419	−.0826	−.0419	−.0827	−.0419
−11.	−.0894	−.0454	−.0895	−.0455	−.0896	−.0455	−.0897	−.0455	−.0818	−.0456	−.0898	−.0456	−.0899	−.0456	−.0900	−.0456	−.0901	−.0457	−.0902	−.0458
−10.	−.0982	−.0499	−.0983	−.0500	−.0984	−.0500	−.0985	−.0501	−.0986	−.0501	−.0987	−.0501	−.0988	−.0502	−.0989	−.0503	−.0990	−.0503	−.0991	−.0504
−9.	−.1089	−.0554	−.1090	−.0555	−.1092	−.0556	−.1093	−.0556	−.1094	−.0557	−.1095	−.0558	−.1096	−.0558	−.1098	−.0559	−.1099	−.0560	−.1100	−.0560
−8.	−.1222	−.0624	−.1224	−.0625	−.1226	−.0625	−.1227	−.0626	−.1229	−.0627	−.1230	−.0628	−.1232	−.0628	−.1233	−.0629	−.1235	−.0630	−.1236	−.0631
−7.	−.1393	−.0713	−.1395	−.0714	−.1397	−.0715	−.1399	−.0716	−.1401	−.0717	−.1403	−.0718	−.1405	−.0719	−.1407	−.0720	−.1409	−.0721	−.1411	−.0722
−6.	−.1619	−.0832	−.1622	−.0833	−.1624	−.0834	−.1627	−.0836	−.1630	−.0837	−.1632	−.0838	−.1635	−.0840	−.1638	−.0841	−.1640	−.0843	−.1643	−.0844
−5.	−.1933	−.0998	−.1937	−.1000	−.1940	−.1002	−.1944	−.1003	−.1948	−.1005	−.1952	−.1007	−.1955	−.1009	−.1959	−.1011	−.1963	−.1013	−.1967	−.1016
−4.	−.2398	−.1247	−.2404	−.1250	−.2410	−.1253	−.2415	−.1256	−.2421	−.1259	−.2427	−.1263	−.2433	−.1266	−.2439	−.1269	−.2445	−.1273	−.2451	−.1276
−3.	−.3161	−.1664	−.3171	−.1669	−.3181	−.1675	−.3191	−.1680	−.3202	−.1686	−.3212	−.1692	−.3223	−.1698	−.3233	−.1703	−.3244	−.1709	−.3254	−.1715
−2.	−.4648	−.2504	−.4670	−.2516	−.4693	−.2529	−.4715	−.2542	−.4738	−.2555	−.4761	−.2569	−.4784	−.2582	−.4807	−.2596	−.4831	−.2609	−.4855	−.2623
−1.	−.8958	−.5139	−.9045	−.5195	−.9133	−.5252	−.9224	−.5311	−.9316	−.5371	−.9410	−.5432	−.9506	−.5495	−.9605	−.5559	−.9705	−.5625	−.9808	−.5693
−0.XX	−.8001	.4479	−.7538	.4874	−.7082	.5255	−.6633	.5622	−.6191	.5976	−.5754	.6318	−.5322	.6647	−.4896	.6965	−.4473	.7271	−.4055	.7567

TABLE B-IV.- CONTINUED

(e) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $59 \geqq XX \geqq 50$

$\frac{x_n-x_o}{\Delta x}$ \ XX	59		58		57		56		55		54		53		52		51		50	
	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
-19.XX	-0.0524	-0.0264	-0.0524	-0.0264	-0.0525	-0.0265	-0.0525	-0.0265	-0.0525	-0.0265	-0.0525	-0.0265	-0.0526	-0.0265	-0.0526	-0.0265	-0.0526	-0.0265	-0.0526	-0.0266
-18.	-.0553	-.0279	-.0553	-.0279	-.0553	-.0279	-.0554	-.0280	-.0554	-.0280	-.0554	-.0280	-.0555	-.0280	-.0555	-.0280	-.0555	-.0280	-.0556	-.0280
-17.	-.0585	-.0296	-.0586	-.0296	-.0586	-.0296	-.0586	-.0296	-.0587	-.0296	-.0587	-.0296	-.0587	-.0297	-.0588	-.0297	-.0588	-.0297	-.0588	-.0297
-16.	-.0622	-.0314	-.0622	-.0314	-.0622	-.0314	-.0623	-.0315	-.0623	-.0315	-.0624	-.0315	-.0624	-.0315	-.0624	-.0315	-.0625	-.0316	-.0625	-.0316
-15.	-.0663	-.0335	-.0663	-.0335	-.0664	-.0336	-.0664	-.0336	-.0665	-.0336	-.0665	-.0336	-.0666	-.0337	-.0666	-.0337	-.0666	-.0337	-.0667	-.0337
-14.	-.0710	-.0359	-.0711	-.0360	-.0711	-.0360	-.0712	-.0360	-.0712	-.0360	-.0713	-.0360	-.0713	-.0361	-.0714	-.0361	-.0714	-.0361	-.0715	-.0362
-13.	-.0764	-.0387	-.0765	-.0387	-.0766	-.0388	-.0766	-.0388	-.0767	-.0388	-.0767	-.0389	-.0768	-.0389	-.0768	-.0389	-.0769	-.0389	-.0770	-.0390
-12.	-.0828	-.0420	-.0828	-.0420	-.0829	-.0421	-.0830	-.0421	-.0830	-.0422	-.0831	-.0422	-.0832	-.0422	-.0832	-.0423	-.0833	-.0423	-.0834	-.0423
-11.	-.0902	-.0458	-.0903	-.0458	-.0904	-.0459	-.0905	-.0459	-.0906	-.0460	-.0906	-.0460	-.0907	-.0461	-.0908	-.0461	-.0909	-.0461	-.0910	-.0462
-10.	-.0992	-.0505	-.0993	-.0505	-.0994	-.0505	-.0995	-.0505	-.0996	-.0506	-.0997	-.0506	-.0998	-.0507	-.0999	-.0507	-.1000	-.0508	-.1001	-.0509
-9.	-.1101	-.0560	-.1102	-.0561	-.1104	-.0562	-.1105	-.0562	-.1106	-.0563	-.1107	-.0564	-.1109	-.0564	-.1110	-.0565	-.1111	-.0566	-.1112	-.0567
-8.	-.1238	-.0631	-.1239	-.0633	-.1241	-.0633	-.1242	-.0634	-.1244	-.0635	-.1245	-.0636	-.1247	-.0636	-.1249	-.0637	-.1250	-.0638	-.1252	-.0639
-7.	-.1413	-.0723	-.1415	-.0724	-.1417	-.0725	-.1419	-.0726	-.1421	-.0727	-.1423	-.0728	-.1425	-.0729	-.1427	-.0730	-.1429	-.0732	-.1431	-.0733
-6.	-.1646	-.0845	-.1648	-.0847	-.1651	-.0848	-.1654	-.0850	-.1657	-.0851	-.1659	-.0853	-.1662	-.0854	-.1665	-.0856	-.1668	-.0857	-.1671	-.0859
-5.	-.1971	-.1018	-.1975	-.1020	-.1979	-.1022	-.1983	-.1024	-.1987	-.1026	-.1991	-.1028	-.1995	-.1030	-.1999	-.1033	-.2003	-.1035	-.2007	-.1037
-4.	-.2457	-.1279	-.2463	-.1282	-.2469	-.1285	-.2476	-.1289	-.2482	-.1292	-.2488	-.1296	-.2494	-.1299	-.2501	-.1302	-.2507	-.1306	-.2513	-.1309
-3.	-.3265	-.1721	-.3276	-.1727	-.3287	-.1733	-.3298	-.1739	-.3309	-.1745	-.3320	-.1751	-.3331	-.1758	-.3342	-.1764	-.3353	-.1770	-.3365	-.1777
-2.	-.4879	-.2637	-.4904	-.2651	-.4928	-.2666	-.4953	-.2680	-.4978	-.2695	-.5004	-.2710	-.5029	-.2725	-.5055	-.2740	-.5082	-.2755	-.5108	-.2771
-1.	-.9914	-.5763	-1.0021	-.5834	-1.0132	-.5907	-1.0245	-.5982	-1.0361	-.6060	-1.0480	-.6139	-1.0601	-.6220	-1.0726	-.6304	-1.0855	-.6390	-1.0986	-.6479
-0.XX	-.3639	.7853	-.3228	.8128	-.2818	.8393	-.2412	.8649	-.2007	.8896	-.1603	.9134	-.1201	.9363	-.0800	.9584	-.0400	.9796	0	1.0000

TABLE B-IV.- CONTINUED

(f) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $49 \geqq XX \geqq 40$

$\frac{x_n-x_o}{\Delta x}$ \ XX	49		48		47		46		45		44		43		42		41		40	
	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
-19.XX	-0.0527	-0.0266	-0.0527	-0.0266	-0.0527	-0.0266	-0.0528	-0.0266	-0.0528	-0.0266	-0.0528	-0.0266	-0.0528	-0.0267	-0.0529	-0.0267	-0.0529	-0.0267	-0.0529	-0.0267
-18.	-.0556	-.0281	-.0556	-.0281	-.0557	-.0281	-.0557	-.0281	-.0557	-.0281	-.0558	-.0281	-.0558	-.0281	-.0558	-.0282	-.0558	-.0282	-.0559	-.0282
-17.	-.0589	-.0297	-.0589	-.0297	-.0589	-.0298	-.0590	-.0298	-.0590	-.0298	-.0590	-.0298	-.0591	-.0298	-.0591	-.0299	-.0592	-.0299	-.0592	-.0299
-16.	-.0626	-.0316	-.0626	-.0316	-.0626	-.0316	-.0627	-.0317	-.0627	-.0317	-.0628	-.0317	-.0628	-.0317	-.0628	-.0318	-.0629	-.0318	-.0629	-.0318
-15.	-.0667	-.0337	-.0668	-.0338	-.0668	-.0338	-.0669	-.0338	-.0669	-.0338	-.0670	-.0338	-.0670	-.0339	-.0670	-.0339	-.0671	-.0339	-.0671	-.0339
-14.	-.0715	-.0362	-.0716	-.0362	-.0716	-.0362	-.0717	-.0363	-.0717	-.0363	-.0718	-.0363	-.0718	-.0363	-.0719	-.0364	-.0719	-.0364	-.0720	-.0364
-13.	-.0770	-.0390	-.0771	-.0390	-.0771	-.0391	-.0772	-.0391	-.0773	-.0391	-.0773	-.0392	-.0774	-.0392	-.0774	-.0392	-.0775	-.0393	-.0776	-.0393
-12.	-.0834	-.0423	-.0835	-.0424	-.0836	-.0424	-.0837	-.0424	-.0837	-.0424	-.0838	-.0425	-.0839	-.0425	-.0839	-.0426	-.0840	-.0426	-.0841	-.0426
-11.	-.0910	-.0462	-.0911	-.0462	-.0912	-.0463	-.0913	-.0463	-.0914	-.0464	-.0915	-.0465	-.0916	-.0465	-.0916	-.0465	-.0917	-.0465	-.0918	-.0466
-10.	-.1002	-.0509	-.1003	-.0510	-.1004	-.0511	-.1005	-.0511	-.1006	-.0511	-.1007	-.0512	-.1008	-.0512	-.1009	-.0513	-.1010	-.0513	-.1011	-.0514
-9.	-.1114	-.0567	-.1115	-.0567	-.1116	-.0568	-.1117	-.0569	-.1118	-.0569	-.1120	-.0570	-.1121	-.0571	-.1122	-.0572	-.1124	-.0573	-.1125	-.0573
-8.	-.1253	-.0640	-.1255	-.0641	-.1256	-.0641	-.1258	-.0642	-.1260	-.0643	-.1261	-.0644	-.1263	-.0645	-.1264	-.0645	-.1266	-.0646	-.1268	-.0648
-7.	-.1433	-.0734	-.1435	-.0735	-.1437	-.0736	-.1439	-.0737	-.1441	-.0738	-.1443	-.0739	-.1446	-.0740	-.1448	-.0741	-.1450	-.0742	-.1452	-.0743
-6.	-.1673	-.0860	-.1676	-.0861	-.1679	-.0863	-.1682	-.0864	-.1685	-.0866	-.1687	-.0867	-.1690	-.0869	-.1693	-.0870	-.1696	-.0872	-.1699	-.0874
-5.	-.2011	-.1039	-.2015	-.1041	-.2019	-.1043	-.2023	-.1046	-.2027	-.1048	-.2031	-.1050	-.2035	-.1052	-.2040	-.1054	-.2044	-.1057	-.2048	-.1059
-4.	-.2519	-.1312	-.2526	-.1316	-.2532	-.1319	-.2539	-.1323	-.2545	-.1327	-.2552	-.1330	-.2558	-.1334	-.2565	-.1337	-.2572	-.1341	-.2578	-.1345
-3.	-.3376	-.1783	-.3388	-.1789	-.3399	-.1796	-.3411	-.1802	-.3423	-.1809	-.3435	-.1816	-.3447	-.1822	-.3459	-.1829	-.3471	-.1836	-.3483	-.1843
-2.	-.5135	-.2786	-.5162	-.2802	-.5190	-.2818	-.5217	-.2834	-.5245	-.2851	-.5274	-.2867	-.5302	-.2884	-.5331	-.2901	-.5360	-.2918	-.5390	-.2936
-1.	-1.1121	-.6571	-1.1260	-.6665	-1.1403	-.6762	-1.1550	-.6862	-1.1701	-.6966	-1.1856	-.7073	-1.2017	-.7184	-1.2182	-.7298	-1.2352	-.7416	-1.2528	-.7539
-0.XX	.0400	1.0196	.0800	1.0384	.1202	1.0565	.1603	1.0738	.2007	1.0903	.2411	1.1061	.2819	1.1212	.3227	1.1355	.3639	1.1492	.4055	1.1622

TABLE B-IV.- CONTINUED

(g) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $39 \geqq XX \geqq 30$

$\frac{x_n-x_o}{\Delta x}$ \ XX	39 j_{no}	39 j_{no}^*	38 j_{no}	38 j_{no}^*	37 j_{no}	37 j_{no}^*	36 j_{no}	36 j_{no}^*	35 j_{no}	35 j_{no}^*
-19.XX	-0.0530	-0.0267	-0.0530	-0.0267	-0.0530	-0.0267	-0.0530	-0.0267	-0.0531	-0.0268
-18.	-.0559	-.0282	-.0559	-.0282	-.0560	-.0283	-.0560	-.0283	-.0560	-.0283
-17.	-.0592	-.0299	-.0593	-.0299	-.0593	-.0299	-.0593	-.0300	-.0594	-.0300
-16.	-.0630	-.0318	-.0630	-.0318	-.0630	-.0319	-.0631	-.0319	-.0631	-.0319
-15.	-.0672	-.0340	-.0672	-.0340	-.0673	-.0340	-.0673	-.0340	-.0674	-.0341
-14.	-.0720	-.0364	-.0721	-.0365	-.0721	-.0365	-.0722	-.0365	-.0722	-.0366
-13.	-.0776	-.0393	-.0777	-.0393	-.0777	-.0394	-.0778	-.0394	-.0779	-.0394
-12.	-.0842	-.0427	-.0842	-.0427	-.0843	-.0427	-.0844	-.0428	-.0844	-.0428
-11.	-.0919	-.0467	-.0920	-.0467	-.0921	-.0467	-.0921	-.0468	-.0922	-.0469
-10.	-.1012	-.0515	-.1013	-.0515	-.1014	-.0515	-.1015	-.0516	-.1016	-.0517
-9.	-.1126	-.0574	-.1127	-.0574	-.1129	-.0575	-.1130	-.0576	-.1131	-.0576
-8.	-.1269	-.0648	-.1271	-.0649	-.1272	-.0650	-.1274	-.0651	-.1276	-.0651
-7.	-.1454	-.0744	-.1456	-.0746	-.1458	-.0747	-.1460	-.0748	-.1462	-.0749
-6.	-.1702	-.0875	-.1705	-.0876	-.1708	-.0878	-.1711	-.0880	-.1714	-.0882
-5.	-.2052	-.1061	-.2056	-.1063	-.2061	-.1066	-.2065	-.1068	-.2069	-.1070
-4.	-.2585	-.1348	-.2592	-.1352	-.2599	-.1356	-.2605	-.1359	-.2612	-.1363
-3.	-.3495	-.1849	-.3508	-.1856	-.3520	-.1863	-.3533	-.1870	-.3545	-.1877
-2.	-.5420	-.2953	-.5450	-.2971	-.5481	-.2990	-.5512	-.3008	-.5543	-.3026
-1.	-1.2709	-.7666	-1.2897	-.7797	-1.3091	-.7934	-1.3291	-.8076	-1.3499	-.8224
-0.XX	.4473	1.1475	.4896	1.1860	.5322	1.1969	.5754	1.2071	.6191	1.2167

$\frac{x_n-x_o}{\Delta x}$ \ XX	34 j_{no}	34 j_{no}^*	33 j_{no}	33 j_{no}^*	32 j_{no}	32 j_{no}^*	31 j_{no}	31 j_{no}^*	30 j_{no}	30 j_{no}^*
-19.XX	-0.0531	-0.0268	-0.0531	-0.0268	-0.0531	-0.0268	-0.0532	-0.0268	-0.0532	-0.0268
-18.	-.0561	-.0283	-.0561	-.0283	-.0561	-.0283	-.0562	-.0283	-.0562	-.0284
-17.	-.0594	-.0300	-.0594	-.0300	-.0595	-.0300	-.0595	-.0301	-.0595	-.0301
-16.	-.0632	-.0319	-.0632	-.0319	-.0632	-.0319	-.0633	-.0320	-.0633	-.0320
-15.	-.0674	-.0341	-.0675	-.0341	-.0675	-.0341	-.0675	-.0342	-.0676	-.0342
-14.	-.0723	-.0366	-.0723	-.0366	-.0724	-.0366	-.0724	-.0367	-.0725	-.0367
-13.	-.0779	-.0395	-.0780	-.0395	-.0780	-.0395	-.0781	-.0396	-.0782	-.0396
-12.	-.0845	-.0428	-.0846	-.0429	-.0847	-.0429	-.0847	-.0430	-.0848	-.0430
-11.	-.0923	-.0469	-.0924	-.0469	-.0925	-.0470	-.0926	-.0470	-.0927	-.0470
-10.	-.1017	-.0518	-.1018	-.0518	-.1019	-.0519	-.1020	-.0519	-.1021	-.0520
-9.	-.1132	-.0577	-.1134	-.0578	-.1135	-.0578	-.1136	-.0579	-.1138	-.0579
-8.	-.1277	-.0652	-.1279	-.0653	-.1281	-.0654	-.1282	-.0655	-.1284	-.0656
-7.	-.1465	-.0750	-.1467	-.0751	-.1469	-.0752	-.1471	-.0753	-.1473	-.0755
-6.	-.1717	-.0883	-.1720	-.0885	-.1722	-.0886	-.1725	-.0888	-.1728	-.0889
-5.	-.2073	-.1072	-.2078	-.1075	-.2082	-.1077	-.2087	-.1080	-.2091	-.1082
-4.	-.2619	-.1367	-.2626	-.1370	-.2633	-.1374	-.2640	-.1378	-.2647	-.1382
-3.	-.3558	-.1884	-.3571	-.1891	-.3584	-.1899	-.3597	-.1906	-.3610	-.1913
-2.	-.5575	-.3045	-.5607	-.3064	-.5639	-.3083	-.5672	-.3103	-.5705	-.3123
-1.	-1.3715	-.8378	-1.3938	-.8538	-1.4171	-.8705	-1.4412	-.8880	-1.4663	-.9062
-0.XX	.6633	1.2255	.7082	1.2337	.7538	1.2412	.8001	1.2480	.8473	1.2542

TABLE B-IV.- CONTINUED

(h) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $29 \geqq XX \geqq 20$

$\frac{x_n-x_o}{\Delta x}$ \ XX	29 j_{no}	29 j_{no}^*	28 j_{no}	28 j_{no}^*	27 j_{no}	27 j_{no}^*	26 j_{no}	26 j_{no}^*	25 j_{no}	25 j_{no}^*
-19.XX	-0.0532	-0.0269	-0.0532	-0.0269	-0.0533	-0.0269	-0.0533	-0.0269	-0.0533	-0.0269
-18.	-.0562	-.0284	-.0563	-.0284	-.0563	-.0284	-.0563	-.0284	-.0564	-.0284
-17.	-.0596	-.0301	-.0596	-.0301	-.0596	-.0301	-.0597	-.0301	-.0597	-.0302
-16.	-.0634	-.0320	-.0634	-.0320	-.0634	-.0321	-.0635	-.0321	-.0635	-.0321
-15.	-.0676	-.0342	-.0677	-.0342	-.0677	-.0342	-.0678	-.0343	-.0678	-.0343
-14.	-.0725	-.0367	-.0726	-.0367	-.0727	-.0368	-.0727	-.0368	-.0728	-.0368
-13.	-.0782	-.0396	-.0783	-.0397	-.0784	-.0397	-.0784	-.0397	-.0785	-.0398
-12.	-.0849	-.0430	-.0849	-.0430	-.0850	-.0431	-.0851	-.0431	-.0852	-.0432
-11.	-.0927	-.0470	-.0928	-.0471	-.0929	-.0472	-.0930	-.0472	-.0931	-.0473
-10.	-.1022	-.0520	-.1023	-.0521	-.1024	-.0521	-.1025	-.0521	-.1027	-.0522
-9.	-.1139	-.0580	-.1140	-.0581	-.1142	-.0582	-.1143	-.0583	-.1144	-.0583
-8.	-.1285	-.0656	-.1287	-.0657	-.1289	-.0658	-.1290	-.0659	-.1292	-.0660
-7.	-.1475	-.0756	-.1478	-.0757	-.1480	-.0758	-.1482	-.0759	-.1484	-.0760
-6.	-.1731	-.0891	-.1734	-.0892	-.1737	-.0894	-.1740	-.0895	-.1744	-.0897
-5.	-.2095	-.1084	-.2100	-.1086	-.2104	-.1089	-.2109	-.1091	-.2113	-.1094
-4.	-.2654	-.1386	-.2661	-.1390	-.2668	-.1393	-.2675	-.1397	-.2683	-.1401
-3.	-.3623	-.1921	-.3637	-.1928	-.3650	-.1936	-.3664	-.1943	-.3677	-.1951
-2.	-.5739	-.3142	-.5773	-.3163	-.5808	-.3183	-.5843	-.3204	-.5878	-.3225
-1.	-1.4925	-.9254	-1.5198	-.9454	-1.5483	-.9677	-1.5782	-.9885	-1.6094	-1.0118
-0.XX	.8954	1.2597	.9445	1.2644	.9946	1.2685	1.0460	1.2720	1.0986	1.2747

$\frac{x_n-x_o}{\Delta x}$ \ XX	24 j_{no}	24 j_{no}^*	23 j_{no}	23 j_{no}^*	22 j_{no}	22 j_{no}^*	21 j_{no}	21 j_{no}^*	20 j_{no}	20 j_{no}^*
-19.XX	-0.0534	-0.0269	-0.0534	-0.0269	-0.0534	-0.0270	-0.0534	-0.0270	-0.0535	-0.0270
-18.	-.0564	-.0285	-.0564	-.0285	-.0564	-.0285	-.0565	-.0285	-.0565	-.0285
-17.	-.0598	-.0302	-.0598	-.0302	-.0598	-.0302	-.0599	-.0302	-.0599	-.0302
-16.	-.0636	-.0321	-.0636	-.0321	-.0636	-.0322	-.0637	-.0322	-.0637	-.0322
-15.	-.0679	-.0343	-.0679	-.0343	-.0680	-.0344	-.0680	-.0344	-.0681	-.0344
-14.	-.0728	-.0369	-.0729	-.0369	-.0729	-.0369	-.0730	-.0369	-.0730	-.0370
-13.	-.0785	-.0398	-.0786	-.0398	-.0787	-.0398	-.0787	-.0399	-.0788	-.0399
-12.	-.0852	-.0432	-.0853	-.0433	-.0854	-.0433	-.0855	-.0433	-.0855	-.0434
-11.	-.0932	-.0473	-.0933	-.0473	-.0934	-.0474	-.0934	-.0475	-.0935	-.0475
-10.	-.1028	-.0523	-.1029	-.0523	-.1030	-.0524	-.1031	-.0524	-.1032	-.0525
-9.	-.1145	-.0584	-.1147	-.0585	-.1148	-.0585	-.1149	-.0586	-.1151	-.0586
-8.	-.1294	-.0661	-.1295	-.0662	-.1297	-.0662	-.1299	-.0663	-.1301	-.0664
-7.	-.1486	-.0761	-.1489	-.0763	-.1491	-.0763	-.1493	-.0765	-.1495	-.0766
-6.	-.1747	-.0899	-.1750	-.0900	-.1753	-.0902	-.1756	-.0904	-.1759	-.0905
-5.	-.2118	-.1096	-.2122	-.1098	-.2127	-.1101	-.2131	-.1104	-.2136	-.1106
-4.	-.2690	-.1404	-.2697	-.1409	-.2705	-.1413	-.2712	-.1417	-.2719	-.1421
-3.	-.3691	-.1959	-.3705	-.1967	-.3719	-.1974	-.3733	-.1982	-.3747	-.1990
-2.	-.5914	-.3247	-.5950	-.3268	-.5987	-.3290	-.6024	-.3312	-.6061	-.3335
-1.	-1.6422	-1.0364	-1.6767	-1.0623	-1.7130	-1.0898	-1.7513	-1.1190	-1.7918	-1.1501
-0.XX	1.1527	1.2766	1.2083	1.2779	1.2657	1.2784	1.3249	1.2782	1.3863	1.2773

TABLE B-IV.- CONTINUED

(i) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $19 \geqq XX \geqq 10$

XX / $\frac{x_n-x_o}{\Delta x}$	19		18		17		16		15		14		13		12		11		10	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
-19.XX	-0.0535	-0.0270	-0.0535	-0.0270	-0.0536	-0.0270	-0.0536	-0.0270	-0.0536	-0.0271	-0.0537	-0.0271	-0.0537	-0.0271	-0.0537	-0.0271	-0.0537	-0.0271	-0.0538	-0.0271
-18.	-.0565	-.0285	-.0566	-.0286	-.0566	-.0286	-.0566	-.0286	-.0567	-.0286	-.0567	-.0286	-.0567	-.0286	-.0568	-.0287	-.0568	-.0287	-.0568	-.0287
-17.	-.0599	-.0303	-.0600	-.0303	-.0600	-.0303	-.0600	-.0303	-.0601	-.0303	-.0601	-.0304	-.0602	-.0304	-.0602	-.0304	-.0602	-.0304	-.0603	-.0304
-16.	-.0638	-.0322	-.0638	-.0322	-.0638	-.0323	-.0639	-.0323	-.0639	-.0323	-.0640	-.0323	-.0640	-.0323	-.0640	-.0324	-.0641	-.0324	-.0641	-.0324
-15.	-.0681	-.0344	-.0681	-.0345	-.0682	-.0345	-.0682	-.0345	-.0683	-.0345	-.0683	-.0345	-.0684	-.0346	-.0684	-.0346	-.0685	-.0346	-.0685	-.0347
-14.	-.0731	-.0370	-.0731	-.0370	-.0732	-.0370	-.0732	-.0371	-.0733	-.0371	-.0733	-.0371	-.0734	-.0372	-.0735	-.0372	-.0735	-.0372	-.0736	-.0372
-13.	-.0788	-.0399	-.0789	-.0400	-.0790	-.0400	-.0790	-.0400	-.0791	-.0401	-.0792	-.0401	-.0792	-.0401	-.0793	-.0402	-.0793	-.0402	-.0794	-.0402
-12.	-.0856	-.0434	-.0857	-.0434	-.0857	-.0435	-.0858	-.0436	-.0859	-.0436	-.0860	-.0436	-.0860	-.0436	-.0861	-.0437	-.0862	-.0437	-.0863	-.0437
-11.	-.0936	-.0475	-.0937	-.0476	-.0938	-.0476	-.0939	-.0477	-.0940	-.0477	-.0941	-.0478	-.0941	-.0478	-.0942	-.0479	-.0943	-.0479	-.0944	-.0480
-10.	-.1033	-.0525	-.1034	-.0526	-.1035	-.0526	-.1036	-.0527	-.1037	-.0528	-.1038	-.0528	-.1039	-.0529	-.1040	-.0529	-.1042	-.0530	-.1043	-.0530
-9.	-.1152	-.0587	-.1153	-.0588	-.1155	-.0589	-.1156	-.0589	-.1157	-.0590	-.1159	-.0591	-.1160	-.0591	-.1161	-.0592	-.1163	-.0592	-.1164	-.0594
-8.	-.1302	-.0665	-.1304	-.0666	-.1306	-.0667	-.1307	-.0668	-.1309	-.0669	-.1311	-.0670	-.1312	-.0670	-.1314	-.0672	-.1316	-.0673	-.1318	-.0674
-7.	-.1498	-.0767	-.1500	-.0769	-.1502	-.0770	-.1504	-.0771	-.1507	-.0772	-.1509	-.0773	-.1511	-.0774	-.1513	-.0776	-.1516	-.0777	-.1518	-.0778
-6.	-.1762	-.0907	-.1765	-.0909	-.1768	-.0910	-.1771	-.0912	-.1775	-.0914	-.1778	-.0915	-.1781	-.0917	-.1784	-.0918	-.1787	-.0920	-.1790	-.0922
-5.	-.2140	-.1108	-.2145	-.1111	-.2150	-.1113	-.2154	-.1116	-.2159	-.1118	-.2164	-.1121	-.2168	-.1123	-.2173	-.1126	-.2178	-.1129	-.2183	-.1131
-4.	-.2727	-.1425	-.2734	-.1430	-.2742	-.1434	-.2749	-.1438	-.2757	-.1442	-.2765	-.1446	-.2772	-.1450	-.2780	-.1454	-.2788	-.1459	-.2796	-.1463
-3.	-.3761	-.1998	-.3776	-.2006	-.3790	-.2014	-.3805	-.2023	-.3819	-.2031	-.3834	-.2039	-.3849	-.2048	-.3864	-.2056	-.3879	-.2065	-.3895	-.2073
-2.	-.6100	-.3358	-.6138	-.3381	-.6177	-.3405	-.6217	-.3428	-.6257	-.3453	-.6298	-.3477	-.6339	-.3502	-.6381	-.3527	-.6423	-.3553	-.6466	-.3579
-1.	-1.8347	-1.1833	-1.8803	-1.2188	-1.9290	-1.2569	-1.9810	-1.2980	-2.0369	-1.3424	-2.0971	-1.3907	-2.1624	-1.4436	-2.2336	-1.5016	-2.3116	-1.5659	-2.3979	-1.6377
-0.XX	1.4500	1.2755	1.5164	1.2729	1.5856	1.2696	1.6582	1.2653	1.7346	1.2602	1.8153	1.2541	1.9010	1.2471	1.9924	1.2391	2.0908	1.2300	2.1972	1.2197

TABLE B-IV.- CONCLUDED

(j) $-19.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.XX$ where $09 \geqq XX \geqq 00$

XX / $\frac{x_n-x_o}{\Delta x}$	09		08		07		06		05		04		03		02		01		00	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
-19.XX	-0.0538	-0.0272	-0.0539	-0.0272	-0.0539	-0.0272	-0.0539	-0.0272	-0.0539	-0.0272	-0.0540	-0.0272	-0.0540	-0.0272	-0.0540	-0.0273	-0.0540	-0.0273	-0.0541	-0.0273
-18.	-.0569	-.0287	-.0569	-.0287	-.0569	-.0287	-.0570	-.0288	-.0570	-.0288	-.0570	-.0288	-.0571	-.0288	-.0571	-.0288	-.0571	-.0288	-.0572	-.0289
-17.	-.0603	-.0305	-.0603	-.0305	-.0604	-.0305	-.0604	-.0305	-.0604	-.0305	-.0605	-.0305	-.0605	-.0306	-.0606	-.0306	-.0606	-.0306	-.0606	-.0306
-16.	-.0642	-.0324	-.0642	-.0324	-.0642	-.0325	-.0643	-.0325	-.0643	-.0325	-.0644	-.0325	-.0644	-.0325	-.0645	-.0326	-.0645	-.0326	-.0645	-.0326
-15.	-.0686	-.0347	-.0686	-.0347	-.0687	-.0347	-.0687	-.0348	-.0688	-.0348	-.0688	-.0348	-.0688	-.0348	-.0689	-.0348	-.0689	-.0349	-.0690	-.0349
-14.	-.0736	-.0373	-.0737	-.0373	-.0737	-.0373	-.0738	-.0373	-.0738	-.0374	-.0739	-.0374	-.0739	-.0374	-.0740	-.0375	-.0741	-.0375	-.0741	-.0375
-13.	-.0795	-.0402	-.0795	-.0403	-.0796	-.0403	-.0797	-.0404	-.0797	-.0404	-.0798	-.0404	-.0799	-.0405	-.0799	-.0405	-.0800	-.0405	-.0800	-.0405
-12.	-.0863	-.0438	-.0864	-.0438	-.0865	-.0439	-.0866	-.0439	-.0866	-.0440	-.0867	-.0440	-.0868	-.0440	-.0869	-.0440	-.0869	-.0441	-.0870	-.0441
-11.	-.0945	-.0480	-.0946	-.0481	-.0947	-.0481	-.0948	-.0482	-.0949	-.0482	-.0949	-.0482	-.0950	-.0483	-.0951	-.0483	-.0952	-.0484	-.0953	-.0484
-10.	-.1044	-.0531	-.1045	-.0531	-.1046	-.0532	-.1047	-.0533	-.1048	-.0533	-.1049	-.0534	-.1050	-.0534	-.1051	-.0535	-.1053	-.0536	-.1054	-.0536
-9.	-.1165	-.0594	-.1167	-.0595	-.1168	-.0596	-.1170	-.0596	-.1171	-.0597	-.1172	-.0598	-.1174	-.0598	-.1175	-.0599	-.1176	-.0599	-.1178	-.0600
-8.	-.1319	-.0674	-.1321	-.0675	-.1323	-.0676	-.1325	-.0677	-.1326	-.0678	-.1328	-.0679	-.1330	-.0680	-.1332	-.0681	-.1334	-.0682	-.1335	-.0683
-7.	-.1520	-.0779	-.1523	-.0780	-.1525	-.0782	-.1527	-.0783	-.1530	-.0784	-.1532	-.0785	-.1534	-.0787	-.1537	-.0788	-.1539	-.0789	-.1542	-.0791
-6.	-.1794	-.0924	-.1797	-.0925	-.1800	-.0927	-.1803	-.0929	-.1807	-.0931	-.1810	-.0932	-.1813	-.0934	-.1817	-.0936	-.1820	-.0937	-.1823	-.0939
-5.	-.2187	-.1134	-.2192	-.1136	-.2197	-.1139	-.2202	-.1141	-.2207	-.1144	-.2212	-.1146	-.2217	-.1149	-.2222	-.1152	-.2226	-.1155	-.2231	-.1157
-4.	-.2804	-.1467	-.2812	-.1472	-.2820	-.1476	-.2828	-.1480	-.2836	-.1485	-.2844	-.1489	-.2852	-.1494	-.2860	-.1498	-.2869	-.1503	-.2877	-.1507
-3.	-.3910	-.2082	-.3926	-.2091	-.3941	-.2100	-.3957	-.2109	-.3973	-.2118	-.3989	-.2127	-.4005	-.2136	-.4022	-.2145	-.4038	-.2155	-.4055	-.2164
-2.	-.6510	-.3606	-.6554	-.3633	-.6599	-.3660	-.6644	-.3687	-.6690	-.3716	-.6737	-.3744	-.6785	-.3773	-.6833	-.3803	-.6882	-.3833	-.6931	-.3863
-1.	-2.4941	-1.7186	-2.6027	-1.8109	-2.7269	-1.9178	-2.8717	-2.0440	-3.0445	-2.1967	-3.2581	-2.3884	-3.5361	-2.6422	-3.9318	-3.0127	-4.6151	-3.6613	$-\infty$	$-\infty$
-0.XX	2.3136	1.2082	2.4423	1.1954	2.5867	1.1811	2.7516	1.1651	2.9444	1.1472	3.1781	1.1271	3.4761	1.1043	3.8918	1.0778	4.5951	1.0460	∞	1.0000

TABLE B-V.— VALUES OF J_{no} AND J_{no}^* USED IN EVALUATING EQUATION (26)

(a) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $00 \leqq XX \leqq 09$

$\frac{x_n-x_o}{\Delta x}$ \ XX	00		01		02		03		04		05		06		07		08		09	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
0.XX	∞	1.0000	4.6151	0.9538	3.9318	0.9214	3.5361	0.8939	3.2581	0.8697	3.0445	0.8478	2.8717	0.8277	2.7269	0.8091	2.6027	0.7918	2.4941	0.7755
1.	.6931	.3069	.6882	.3049	.6833	.3030	.6785	.3012	.6737	.2993	.6690	.2975	.6644	.2957	.6599	.2939	.6554	.2922	.6510	.2904
2.	.4055	.1891	.4038	.1884	.4022	.1876	.4005	.1869	.3989	.1862	.3973	.1855	.3957	.1848	.3941	.1842	.3926	.1835	.3910	.1828
3.	.2877	.1370	.2869	.1366	.2860	.1362	.2852	.1358	.2844	.1355	.2836	.1351	.2828	.1347	.2820	.1344	.2812	.1340	.2804	.1337
4.	.2231	.1074	.2226	.1072	.2222	.1070	.2217	.1067	.2212	.1065	.2207	.1063	.2202	.1060	.2197	.1058	.2192	.1056	.2187	.1054
5.	.1823	.0884	.1820	.0882	.1817	.0881	.1813	.0879	.1810	.0878	.1806	.0876	.1803	.0875	.1800	.0873	.1797	.0872	.1794	.0870
6.	.1542	.0751	.1539	.0750	.1537	.0749	.1534	.0748	.1532	.0747	.1530	.0745	.1527	.0744	.1525	.0743	.1523	.0742	.1520	.0741
7.	.1335	.0653	.1334	.0652	.1332	.0651	.1330	.0650	.1328	.0649	.1326	.0649	.1325	.0648	.1323	.0647	.1321	.0646	.1319	.0645
8.	.1178	.0577	.1176	.0577	.1175	.0576	.1174	.0576	.1172	.0575	.1171	.0574	.1170	.0573	.1168	.0572	.1167	.0572	.1165	.0571
9.	.1054	.0518	.1053	.0517	.1051	.0517	.1050	.0516	.1049	.0515	.1048	.0515	.1047	.0514	.1046	.0514	.1045	.0513	.1044	.0513
10.	.0953	.0469	.0952	.0469	.0951	.0468	.0950	.0468	.0949	.0467	.0949	.0467	.0948	.0466	.0947	.0466	.0946	.0465	.0945	.0465
11.	.0870	.0429	.0869	.0428	.0869	.0428	.0868	.0428	.0867	.0427	.0866	.0427	.0866	.0427	.0865	.0427	.0864	.0426	.0863	.0426
12.	.0800	.0395	.0800	.0395	.0799	.0395	.0799	.0394	.0798	.0393	.0797	.0393	.0797	.0393	.0796	.0393	.0795	.0393	.0795	.0392
13.	.0741	.0366	.0741	.0366	.0740	.0365	.0739	.0365	.0739	.0365	.0738	.0365	.0738	.0364	.0737	.0364	.0737	.0364	.0736	.0364
14.	.0690	.0341	.0689	.0341	.0689	.0340	.0688	.0340	.0688	.0340	.0688	.0340	.0687	.0340	.0687	.0339	.0686	.0339	.0686	.0339
15.	.0645	.0319	.0645	.0319	.0645	.0319	.0644	.0319	.0644	.0318	.0643	.0318	.0643	.0318	.0642	.0318	.0642	.0318	.0642	.0317
16.	.0606	.0300	.0606	.0300	.0606	.0300	.0605	.0300	.0605	.0299	.0604	.0299	.0604	.0299	.0604	.0299	.0603	.0299	.0603	.0298
17.	.0572	.0283	.0571	.0283	.0571	.0283	.0571	.0283	.0570	.0282	.0570	.0282	.0570	.0282	.0569	.0282	.0569	.0282	.0569	.0282
18.	.0541	.0268	.0540	.0268	.0540	.0268	.0540	.0268	.0540	.0267	.0539	.0267	.0539	.0267	.0539	.0267	.0538	.0267	.0538	.0266
19.XX	.0513	.0254	.0513	.0254	.0512	.0254	.0512	.0254	.0512	.0254	.0512	.0254	.0511	.0253	.0511	.0253	.0511	.0253	.0511	.0253

TABLE B-V. — CONTINUED

(b) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $10 \leqq XX \leqq 19$

$\frac{x_n-x_o}{\Delta x}$ \ XX	10		11		12		13		14		15		16		17		18		19	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
0.XX	2.3979	0.7602	2.3116	0.7457	2.2336	0.7320	2.1624	0.7189	2.0971	0.7064	2.0369	0.6945	1.9810	0.6830	1.9290	0.6721	1.8803	0.6615	1.8347	0.6514
1.	.6466	.2887	.6423	.2870	.6381	.2853	.6339	.2837	.6298	.2821	.6257	.2804	.6217	.2788	.6177	.2773	.6138	.2757	.6100	.2742
2.	.3895	.1821	.3879	.1815	.3864	.1808	.3849	.1801	.3834	.1795	.3819	.1788	.3805	.1782	.3790	.1776	.3776	.1769	.3761	.1763
3.	.2796	.1333	.2788	.1329	.2780	.1326	.2772	.1322	.2765	.1319	.2757	.1315	.2749	.1312	.2742	.1308	.2734	.1305	.2727	.1302
4.	.2183	.1052	.2178	.1049	.2173	.1047	.2168	.1045	.2164	.1043	.2159	.1041	.2154	.1039	.2150	.1036	.2145	.1034	.2140	.1032
5.	.1790	.0869	.1787	.0867	.1784	.0866	.1781	.0864	.1778	.0863	.1775	.0861	.1771	.0860	.1768	.0858	.1765	.0857	.1762	.0855
6.	.1518	.0740	.1516	.0739	.1513	.0738	.1511	.0736	.1509	.0735	.1507	.0734	.1504	.0733	.1502	.0732	.1500	.0731	.1498	.0730
7.	.1318	.0644	.1316	.0643	.1314	.0643	.1312	.0642	.1311	.0641	.1309	.0640	.1307	.0640	.1306	.0639	.1304	.0638	.1302	.0637
8.	.1164	.0571	.1163	.0570	.1161	.0570	.1160	.0569	.1159	.0568	.1157	.0568	.1156	.0567	.1155	.0566	.1153	.0566	.1152	.0565
9.	.1043	.0512	.1042	.0512	.1040	.0511	.1039	.0511	.1038	.0510	.1037	.0510	.1036	.0509	.1035	.0509	.1034	.0508	.1033	.0508
10.	.0944	.0465	.0943	.0464	.0942	.0464	.0941	.0463	.0941	.0463	.0940	.0463	.0939	.0462	.0938	.0462	.0937	.0461	.0936	.0461
11.	.0863	.0425	.0862	.0425	.0861	.0424	.0860	.0424	.0860	.0424	.0859	.0423	.0858	.0423	.0857	.0422	.0857	.0422	.0856	.0421
12.	.0794	.0392	.0793	.0392	.0793	.0391	.0792	.0391	.0792	.0391	.0791	.0390	.0790	.0390	.0790	.0390	.0789	.0389	.0788	.0389
13.	.0736	.0363	.0735	.0363	.0735	.0363	.0734	.0362	.0733	.0362	.0733	.0362	.0732	.0362	.0732	.0361	.0731	.0361	.0731	.0361
14.	.0685	.0339	.0685	.0338	.0684	.0338	.0684	.0338	.0683	.0338	.0683	.0338	.0682	.0337	.0682	.0337	.0681	.0337	.0681	.0337
15.	.0641	.0317	.0641	.0317	.0640	.0317	.0640	.0317	.0640	.0316	.0639	.0316	.0639	.0316	.0638	.0316	.0638	.0316	.0638	.0315
16.	.0603	.0298	.0602	.0298	.0602	.0298	.0602	.0298	.0601	.0298	.0601	.0297	.0600	.0297	.0600	.0297	.0600	.0297	.0599	.0297
17.	.0568	.0281	.0568	.0281	.0568	.0281	.0567	.0281	.0567	.0281	.0567	.0281	.0566	.0281	.0566	.0280	.0566	.0280	.0565	.0280
18.	.0538	.0266	.0537	.0266	.0537	.0266	.0537	.0266	.0537	.0266	.0536	.0266	.0536	.0266	.0536	.0266	.0535	.0265	.0535	.0265
19.XX	.0510	.0253	.0510	.0253	.0510	.0253	.0510	.0253	.0509	.0252	.0509	.0252	.0509	.0252	.0509	.0252	.0508	.0252	.0508	.0252

TABLE B-V. – CONTINUED

(c) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $20 \leqq XX \leqq 29$

XX	20		21		22		23		24		25		26		27		28		29	
$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
0.XX	1.7918	0.6416	1.7513	0.6322	1.7130	0.6231	1.6767	0.6144	1.6422	0.6059	1.6094	0.5976	1.5782	0.5897	1.5483	0.5819	1.5198	0.5745	1.4925	0.5672
1.	.6061	.2726	.6024	.2711	.5987	.2696	.5950	.2682	.5914	.2667	.5878	.2653	.5843	.2638	.5808	.2624	.5773	.2610	.5739	.2597
2.	.3747	.1757	.3732	.1751	.3719	.1744	.3705	.1738	.3691	.1732	.3677	.1726	.3664	.1720	.3650	.1714	.3637	.1708	.3623	.1703
3.	.2719	.1298	.2712	.1295	.2705	.1291	.2697	.1288	.2690	.1285	.2683	.1282	.2675	.1278	.2668	.1275	.2661	.1272	.2654	.1268
4.	.2136	.1030	.2131	.1028	.2127	.1026	.2122	.1023	.2118	.1021	.2113	.1019	.2109	.1017	.2104	.1015	.2100	.1013	.2095	.1011
5.	.1759	.0854	.1756	.0852	.1753	.0851	.1750	.0849	.1747	.0848	.1744	.0846	.1740	.0845	.1737	.0844	.1734	.0842	.1731	.0841
6.	.1495	.0729	.1493	.0728	.1491	.0727	.1489	.0726	.1486	.0725	.1484	.0724	.1482	.0723	.1480	.0722	.1478	.0721	.1475	.0720
7.	.1301	.0636	.1299	.0635	.1297	.0635	.1295	.0634	.1294	.0633	.1292	.0632	.1290	.0631	.1289	.0631	.1287	.0630	.1285	.0629
8.	.1151	.0564	.1149	.0564	.1148	.0563	.1147	.0562	.1145	.0562	.1144	.0561	.1143	.0560	.1142	.0560	.1140	.0559	.1139	.0558
9.	.1032	.0507	.1031	.0506	.1030	.0506	.1029	.0506	.1028	.0505	.1027	.0504	.1025	.0504	.1024	.0504	.1023	.0503	.1022	.0503
10.	.0935	.0460	.0934	.0460	.0934	.0460	.0933	.0459	.0932	.0459	.0931	.0459	.0930	.0458	.0929	.0458	.0928	.0457	.0927	.0457
11.	.0855	.0421	.0855	.0421	.0854	.0420	.0853	.0420	.0852	.0420	.0852	.0420	.0851	.0419	.0850	.0419	.0849	.0419	.0849	.0418
12.	.0788	.0389	.0787	.0388	.0787	.0388	.0786	.0388	.0785	.0388	.0785	.0387	.0784	.0387	.0783	.0387	.0783	.0386	.0782	.0386
13.	.0730	.0361	.0730	.0360	.0729	.0360	.0729	.0360	.0728	.0360	.0728	.0359	.0727	.0359	.0727	.0359	.0726	.0359	.0725	.0358
14.	.0681	.0336	.0680	.0336	.0680	.0336	.0679	.0336	.0679	.0335	.0678	.0335	.0678	.0335	.0677	.0335	.0677	.0335	.0676	.0334
15.	.0637	.0315	.0637	.0315	.0636	.0315	.0636	.0315	.0636	.0314	.0635	.0314	.0635	.0314	.0634	.0314	.0634	.0314	.0634	.0313
16.	.0599	.0297	.0599	.0296	.0598	.0296	.0598	.0296	.0598	.0296	.0597	.0296	.0597	.0295	.0596	.0295	.0596	.0295	.0596	.0295
17.	.0565	.0280	.0565	.0280	.0564	.0280	.0564	.0279	.0564	.0279	.0564	.0279	.0563	.0279	.0563	.0279	.0563	.0279	.0562	.0279
18.	.0535	.0265	.0535	.0265	.0534	.0265	.0534	.0265	.0534	.0264	.0533	.0264	.0533	.0264	.0533	.0264	.0533	.0264	.0532	.0264
19.XX	.0508	.0252	.0507	.0252	.0507	.0251	.0507	.0251	.0507	.0251	.0506	.0251	.0506	.0251	.0506	.0251	.0506	.0251	.0505	.0251

TABLE B-V. – CONTINUED

(d) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $30 \leqq XX \leqq 39$

XX	30		31		32		33		34		35		36		37		38		39	
$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
0.XX	1.4663	0.5601	1.4412	0.5532	1.4171	0.5465	1.3938	0.5400	1.3715	0.5337	1.3499	0.5275	1.3291	0.5215	1.3091	0.5156	1.2897	0.5099	1.2709	0.5043
1.	.5705	.2583	.5672	.2569	.5639	.2556	.5607	.2543	.5575	.2530	.5543	.2517	.5512	.2504	.5481	.2491	.5450	.2479	.5420	.2466
2.	.3610	.1697	.3597	.1691	.3584	.1685	.3571	.1680	.3558	.1674	.3545	.1668	.3533	.1663	.3520	.1657	.3508	.1652	.3495	.1646
3.	.2647	.1265	.2640	.1262	.2633	.1259	.2626	.1256	.2619	.1252	.2612	.1249	.2605	.1246	.2599	.1243	.2592	.1240	.2585	.1237
4.	.2091	.1009	.2087	.1007	.2082	.1005	.2078	.1003	.2073	.1001	.2069	.0999	.2065	.0997	.2061	.0995	.2056	.0993	.2052	.0991
5.	.1728	.0839	.1725	.0838	.1722	.0837	.1720	.0835	.1717	.0834	.1714	.0832	.1711	.0831	.1708	.0830	.1705	.0828	.1702	.0827
6.	.1473	.0719	.1471	.0718	.1469	.0717	.1467	.0716	.1465	.0715	.1462	.0713	.1460	.0712	.1458	.0711	.1456	.0711	.1454	.0710
7.	.1284	.0628	.1282	.0627	.1281	.0627	.1279	.0626	.1277	.0625	.1276	.0624	.1274	.0623	.1272	.0622	.1271	.0622	.1269	.0621
8.	.1138	.0558	.1136	.0557	.1135	.0557	.1134	.0556	.1132	.0556	.1131	.0555	.1130	.0554	.1129	.0554	.1127	.0553	.1126	.0552
9.	.1021	.0502	.1020	.0502	.1019	.0501	.1018	.0500	.1017	.0500	.1016	.0500	.1015	.0499	.1014	.0499	.1013	.0498	.1012	.0497
10.	.0927	.0456	.0926	.0456	.0925	.0455	.0924	.0455	.0923	.0454	.0922	.0454	.0921	.0454	.0921	.0453	.0920	.0453	.0919	.0452
11.	.0848	.0418	.0847	.0417	.0847	.0417	.0846	.0417	.0845	.0417	.0844	.0416	.0844	.0416	.0843	.0416	.0842	.0416	.0842	.0415
12.	.0782	.0386	.0781	.0385	.0780	.0385	.0780	.0385	.0779	.0385	.0779	.0384	.0778	.0384	.0777	.0384	.0777	.0383	.0776	.0383
13.	.0725	.0358	.0724	.0358	.0724	.0358	.0723	.0357	.0723	.0357	.0722	.0357	.0722	.0357	.0721	.0356	.0721	.0356	.0720	.0356
14.	.0676	.0334	.0675	.0334	.0675	.0334	.0675	.0333	.0674	.0333	.0674	.0333	.0673	.0333	.0673	.0333	.0672	.0332	.0672	.0332
15.	.0633	.0313	.0633	.0313	.0632	.0313	.0632	.0313	.0632	.0312	.0631	.0312	.0631	.0312	.0630	.0312	.0630	.0312	.0629	.0311
16.	.0595	.0295	.0595	.0294	.0595	.0294	.0594	.0294	.0594	.0294	.0594	.0294	.0593	.0294	.0593	.0293	.0593	.0293	.0592	.0293
17.	.0562	.0278	.0562	.0278	.0561	.0278	.0561	.0278	.0561	.0278	.0560	.0278	.0560	.0277	.0560	.0277	.0559	.0277	.0559	.0277
18.	.0532	.0264	.0532	.0264	.0531	.0263	.0531	.0263	.0531	.0263	.0531	.0263	.0530	.0263	.0530	.0263	.0530	.0262	.0530	.0262
19.XX	.0505	.0251	.0505	.0250	.0505	.0250	.0504	.0250	.0504	.0250	.0504	.0250	.0504	.0250	.0503	.0250	.0503	.0249	.0503	.0249

TABLE B-V. – CONTINUED

(e) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $40 \leqq XX \leqq 49$

XX	40		41		42		43		44		45		46		47		48		49	
$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
0.XX	1.2528	0.4989	1.2352	0.4936	1.2182	0.4884	1.2017	0.4833	1.1856	0.4783	1.1701	0.4735	1.1550	0.4687	1.1403	0.4641	1.1260	0.4595	1.1121	0.4551
1.	.5390	.2454	.5360	.2442	.5331	.2430	.5302	.2418	.5274	.2406	.5245	.2394	.5217	.2383	.5190	.2371	.5162	.2360	.5135	.2349
2.	.3483	.1641	.3471	.1635	.3459	.1630	.3447	.1625	.3435	.1619	.3423	.1614	.3411	.1609	.3399	.1604	.3388	.1598	.3376	.1593
3.	.2578	.1234	.2572	.1231	.2565	.1228	.2558	.1225	.2552	.1222	.2545	.1219	.2539	.1216	.2532	.1213	.2526	.1210	.2519	.1207
4.	.2048	.0989	.2044	.0987	.2040	.0985	.2035	.0983	.2031	.0981	.2027	.0979	.2023	.0977	.2019	.0976	.2015	.0974	.2011	.0972
5.	.1699	.0825	.1696	.0824	.1693	.0823	.1690	.0821	.1687	.0820	.1685	.0819	.1682	.0817	.1679	.0816	.1676	.0815	.1673	.0813
6.	.1452	.0708	.1450	.0707	.1448	.0706	.1446	.0705	.1443	.0704	.1441	.0703	.1439	.0702	.1437	.0701	.1435	.0700	.1433	.0699
7.	.1268	.0620	.1266	.0620	.1264	.0619	.1263	.0618	.1261	.0617	.1260	.0616	.1258	.0616	.1256	.0615	.1255	.0614	.1253	.0614
8.	.1125	.0552	.1124	.0552	.1122	.0551	.1121	.0550	.1120	.0550	.1118	.0549	.1117	.0548	.1116	.0548	.1115	.0547	.1114	.0546
9.	.1011	.0497	.1010	.0496	.1009	.0496	.1008	.0496	.1007	.0495	.1006	.0495	.1005	.0494	.1004	.0493	.1003	.0493	.1002	.0493
10.	.0918	.0452	.0917	.0452	.0916	.0451	.0916	.0451	.0915	.0450	.0914	.0450	.0913	.0450	.0912	.0449	.0911	.0449	.0910	.0448
11.	.0841	.0415	.0840	.0414	.0839	.0414	.0839	.0413	.0838	.0413	.0837	.0413	.0837	.0412	.0836	.0412	.0835	.0412	.0834	.0412
12.	.0776	.0383	.0775	.0383	.0774	.0382	.0774	.0382	.0773	.0382	.0773	.0381	.0772	.0381	.0771	.0381	.0771	.0380	.0770	.0380
13.	.0720	.0355	.0719	.0355	.0719	.0355	.0718	.0355	.0718	.0355	.0717	.0354	.0717	.0354	.0716	.0354	.0716	.0354	.0715	.0353
14.	.0671	.0332	.0671	.0332	.0670	.0332	.0670	.0331	.0670	.0331	.0669	.0331	.0669	.0331	.0668	.0330	.0668	.0330	.0667	.0330
15.	.0629	.0311	.0629	.0311	.0628	.0311	.0628	.0311	.0628	.0310	.0627	.0310	.0627	.0310	.0626	.0310	.0626	.0310	.0626	.0310
16.	.0592	.0293	.0592	.0293	.0591	.0293	.0591	.0293	.0590	.0292	.0590	.0292	.0590	.0292	.0589	.0292	.0589	.0292	.0589	.0292
17.	.0559	.0277	.0558	.0277	.0558	.0277	.0558	.0276	.0558	.0276	.0557	.0276	.0557	.0276	.0557	.0276	.0556	.0276	.0556	.0275
18.	.0529	.0262	.0529	.0262	.0529	.0262	.0528	.0262	.0528	.0262	.0528	.0262	.0528	.0261	.0527	.0261	.0527	.0261	.0527	.0261
19.XX	.0503	.0249	.0502	.0249	.0502	.0249	.0502	.0249	.0502	.0249	.0501	.0249	.0501	.0249	.0501	.0248	.0501	.0248	.0500	.0248

TABLE B-V.– CONTINUED

(f) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $50 \leqq XX \leqq 59$

XX	50		51		52		53		54		55		56		57		58		59	
$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
0.XX	1.0986	0.4507	1.0855	0.4464	1.0726	0.4422	1.0601	0.4381	1.0480	0.4341	1.0361	0.4301	1.0245	0.4263	1.0132	0.4225	1.0021	0.4188	0.9914	0.4151
1.	.5108	.2338	.5082	.2327	.5055	.2316	.5029	.2305	.5004	.2294	.4978	.2284	.4953	.2273	.4928	.2263	.4904	.2252	.4879	.2242
2.	.3365	.1588	.3353	.1583	.3342	.1578	.3331	.1573	.3320	.1568	.3309	.1563	.3298	.1558	.3287	.1553	.3276	.1549	.3265	.1544
3.	.2513	.1204	.2507	.1201	.2501	.1198	.2494	.1195	.2488	.1192	.2482	.1190	.2476	.1187	.2469	.1184	.2463	.1181	.2457	.1178
4.	.2007	.0970	.2003	.0968	.1999	.0966	.1995	.0964	.1991	.0962	.1987	.0961	.1983	.0959	.1979	.0957	.1975	.0955	.1971	.0953
5.	.1671	.0812	.1668	.0811	.1665	.0809	.1662	.0808	.1659	.0807	.1657	.0805	.1654	.0804	.1651	.0803	.1648	.0802	.1646	.0800
6.	.1431	.0698	.1429	.0697	.1427	.0697	.1425	.0696	.1423	.0695	.1421	.0694	.1419	.0693	.1417	.0691	.1415	.0690	.1413	.0690
7.	.1252	.0613	.1250	.0612	.1249	.0611	.1247	.0611	.1245	.0610	.1244	.0609	.1242	.0608	.1241	.0608	.1239	.0607	.1238	.0606
8.	.1112	.0546	.1111	.0545	.1110	.0545	.1109	.0544	.1107	.0543	.1106	.0543	.1105	.0542	.1104	.0541	.1102	.0541	.1101	.0541
9.	.1001	.0492	.1000	.0492	.0999	.0491	.0998	.0491	.0997	.0490	.0996	.0490	.0995	.0489	.0994	.0489	.0993	.0488	.0992	.0487
10.	.0910	.0448	.0909	.0447	.0908	.0447	.0907	.0447	.0906	.0446	.0906	.0446	.0905	.0446	.0904	.0445	.0903	.0445	.0902	.0444
11.	.0834	.0411	.0833	.0410	.0832	.0410	.0832	.0410	.0831	.0409	.0830	.0409	.0830	.0409	.0829	.0408	.0828	.0408	.0828	.0408
12.	.0770	.0380	.0769	.0380	.0768	.0379	.0768	.0379	.0767	.0379	.0767	.0378	.0766	.0378	.0765	.0378	.0765	.0378	.0764	.0377
13.	.0715	.0353	.0714	.0353	.0714	.0353	.0713	.0352	.0713	.0352	.0712	.0352	.0712	.0352	.0712	.0351	.0711	.0351	.0710	.0351
14.	.0667	.0330	.0666	.0330	.0666	.0329	.0666	.0329	.0665	.0329	.0665	.0329	.0664	.0329	.0664	.0328	.0663	.0328	.0663	.0328
15.	.0625	.0309	.0625	.0309	.0624	.0309	.0624	.0309	.0624	.0309	.0623	.0308	.0623	.0308	.0622	.0308	.0622	.0308	.0622	.0308
16.	.0588	.0291	.0588	.0291	.0588	.0291	.0587	.0291	.0587	.0291	.0587	.0291	.0586	.0290	.0586	.0290	.0586	.0290	.0585	.0290
17.	.0556	.0275	.0555	.0275	.0555	.0275	.0555	.0275	.0554	.0275	.0554	.0274	.0554	.0274	.0553	.0274	.0553	.0274	.0553	.0274
18.	.0526	.0261	.0526	.0261	.0526	.0261	.0526	.0260	.0525	.0260	.0525	.0260	.0525	.0260	.0525	.0260	.0524	.0260	.0524	.0260
19.XX	.0500	.0248	.0500	.0248	.0500	.0248	.0499	.0248	.0499	.0247	.0499	.0247	.0499	.0247	.0498	.0247	.0498	.0247	.0498	.0247

TABLE B-V. - CONTINUED

(g) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $60 \leqq XX \leqq 69$

$\frac{x_n-x_o}{\Delta x}$ \ XX	60		61		62		63		64		65		66		67		68		69	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
0.XX	0.9808	0.4115	0.9705	0.4080	0.9605	0.4045	0.9506	0.4011	0.9410	0.3978	0.9316	0.3945	0.9224	0.3912	0.9133	0.3881	0.9045	0.3850	0.8958	0.3819
1.	.4855	.2232	.4831	.2222	.4807	.2212	.4784	.2202	.4761	.2192	.4738	.2183	.4715	.2173	.4693	.2163	.4670	.2154	.4648	.2145
2.	.3254	.1539	.3244	.1534	.3233	.1530	.3223	.1525	.3212	.1520	.3202	.1516	.3191	.1511	.3181	.1506	.3171	.1502	.3161	.1497
3.	.2451	.1176	.2445	.1173	.2439	.1170	.2433	.1167	.2427	.1164	.2421	.1162	.2415	.1159	.2410	.1157	.2404	.1154	.2398	.1151
4.	.1967	.0951	.1963	.0950	.1959	.0948	.1955	.0946	.1952	.0944	.1948	.0943	.1944	.0941	.1940	.0939	.1937	.0937	.1933	.0935
5.	.1643	.0799	.1640	.0797	.1638	.0796	.1635	.0795	.1632	.0794	.1630	.0793	.1627	.0791	.1624	.0790	.1622	.0789	.1619	.0788
6.	.1411	.0689	.1409	.0688	.1407	.0687	.1405	.0686	.1403	.0685	.1401	.0684	.1399	.0683	.1397	.0682	.1395	.0681	.1393	.0681
7.	.1236	.0605	.1235	.0605	.1233	.0604	.1232	.0603	.1230	.0602	.1229	.0602	.1227	.0601	.1226	.0600	.1224	.0599	.1222	.0599
8.	.1100	.0540	.1099	.0540	.1098	.0539	.1096	.0539	.1095	.0538	.1094	.0537	.1093	.0537	.1092	.0536	.1090	.0535	.1089	.0535
9.	.0991	.0487	.0990	.0487	.0989	.0486	.0988	.0486	.0987	.0486	.0986	.0485	.0985	.0484	.0984	.0484	.0983	.0483	.0982	.0483
10.	.0902	.0444	.0901	.0444	.0900	.0443	.0899	.0443	.0898	.0443	.0897	.0442	.0897	.0442	.0896	.0441	.0895	.0441	.0894	.0440
11.	.0827	.0408	.0826	.0407	.0826	.0407	.0825	.0407	.0824	.0407	.0823	.0406	.0823	.0406	.0822	.0405	.0822	.0405	.0821	.0404
12.	.0764	.0377	.0763	.0377	.0763	.0376	.0762	.0376	.0761	.0376	.0761	.0376	.0760	.0375	.0760	.0375	.0759	.0375	.0758	.0374
13.	.0710	.0351	.0709	.0350	.0709	.0350	.0708	.0350	.0708	.0350	.0707	.0349	.0707	.0349	.0706	.0349	.0706	.0349	.0705	.0348
14.	.0662	.0328	.0662	.0327	.0662	.0327	.0661	.0327	.0661	.0327	.0660	.0327	.0660	.0326	.0659	.0326	.0659	.0326	.0659	.0326
15.	.0621	.0307	.0621	.0307	.0621	.0307	.0620	.0307	.0620	.0307	.0619	.0306	.0619	.0306	.0619	.0306	.0618	.0306	.0618	.0306
16.	.0585	.0290	.0585	.0289	.0584	.0289	.0584	.0289	.0584	.0289	.0583	.0289	.0583	.0289	.0583	.0288	.0582	.0288	.0582	.0288
17.	.0553	.0274	.0552	.0274	.0552	.0273	.0552	.0273	.0551	.0273	.0551	.0273	.0551	.0273	.0550	.0273	.0550	.0273	.0550	.0273
18.	.0524	.0260	.0523	.0259	.0523	.0259	.0523	.0259	.0523	.0259	.0522	.0259	.0522	.0259	.0522	.0259	.0521	.0259	.0521	.0258
19.XX	.0498	.0247	.0497	.0247	.0497	.0247	.0497	.0246	.0497	.0246	.0496	.0246	.0496	.0246	.0496	.0246	.0496	.0246	.0495	.0246

TABLE B-V. - CONTINUED

(h) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $70 \leqq XX \leqq 79$

$\frac{x_n-x_o}{\Delta x}$ \ XX	70		71		72		73		74		75		76		77		78		79	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
0.XX	0.8873	0.3789	0.8790	0.3759	0.8708	0.3730	0.8628	0.3701	0.8550	0.3673	0.8473	0.3645	0.8398	0.3618	0.8323	0.3591	0.8251	0.3564	0.8179	0.3538
1.	.4626	.2135	.4605	.2126	.4583	.2117	.4562	.2108	.4541	.2099	.4520	.2090	.4499	.2081	.4479	.2073	.4458	.2064	.4438	.2056
2.	.3151	.1493	.3141	.1488	.3131	.1484	.3121	.1480	.3111	.1475	.3102	.1471	.3092	.1466	.3082	.1462	.3073	.1458	.3063	.1454
3.	.2392	.1149	.2387	.1146	.2381	.1143	.2375	.1141	.2370	.1138	.2364	.1135	.2358	.1133	.2353	.1130	.2347	.1128	.2342	.1125
4.	.1929	.0933	.1925	.0932	.1922	.0930	.1918	.0928	.1914	.0927	.1911	.0925	.1907	.0923	.1903	.0921	.1900	.0920	.1896	.0918
5.	.1616	.0786	.1614	.0785	.1611	.0784	.1609	.0783	.1606	.0781	.1603	.0780	.1601	.0779	.1598	.0778	.1596	.0777	.1593	.0776
6.	.1391	.0680	.1389	.0679	.1387	.0678	.1385	.0677	.1383	.0676	.1382	.0675	.1380	.0674	.1378	.0673	.1376	.0672	.1374	.0671
7.	.1221	.0598	.1220	.0598	.1218	.0597	.1217	.0596	.1215	.0595	.1214	.0595	.1212	.0594	.1211	.0593	.1209	.0593	.1208	.0592
8.	.1088	.0534	.1087	.0534	.1086	.0533	.1085	.0532	.1083	.0532	.1082	.0531	.1081	.0530	.1080	.0530	.1079	.0529	.1078	.0529
9.	.0981	.0483	.0980	.0482	.0979	.0482	.0978	.0482	.0977	.0481	.0976	.0481	.0975	.0480	.0974	.0480	.0974	.0479	.0973	.0478
10.	.0893	.0440	.0893	.0440	.0892	.0440	.0891	.0439	.0890	.0439	.0889	.0438	.0889	.0438	.0888	.0437	.0887	.0437	.0886	.0437
11.	.0820	.0404	.0819	.0404	.0819	.0404	.0818	.0404	.0817	.0403	.0817	.0403	.0816	.0403	.0815	.0402	.0815	.0402	.0814	.0401
12.	.0758	.0374	.0757	.0374	.0757	.0374	.0756	.0373	.0756	.0373	.0755	.0373	.0755	.0372	.0754	.0372	.0753	.0372	.0753	.0372
13.	.0705	.0348	.0704	.0348	.0704	.0348	.0703	.0347	.0703	.0347	.0702	.0347	.0702	.0347	.0701	.0346	.0701	.0346	.0700	.0346
14.	.0658	.0325	.0658	.0325	.0657	.0325	.0657	.0325	.0656	.0325	.0656	.0324	.0656	.0324	.0655	.0324	.0655	.0324	.0654	.0324
15.	.0617	.0306	.0617	.0305	.0617	.0305	.0616	.0305	.0616	.0305	.0616	.0305	.0615	.0304	.0615	.0304	.0614	.0304	.0614	.0304
16.	.0582	.0288	.0581	.0288	.0581	.0288	.0581	.0287	.0580	.0287	.0580	.0287	.0580	.0287	.0579	.0287	.0579	.0287	.0578	.0286
17.	.0550	.0272	.0549	.0272	.0549	.0272	.0549	.0272	.0548	.0272	.0548	.0271	.0548	.0271	.0547	.0271	.0547	.0271	.0547	.0271
18.	.0521	.0258	.0521	.0258	.0520	.0258	.0520	.0258	.0520	.0258	.0520	.0258	.0519	.0257	.0519	.0257	.0519	.0257	.0519	.0257
19.XX	.0495	.0246	.0495	.0245	.0495	.0245	.0494	.0245	.0494	.0245	.0494	.0245	.0494	.0245	.0493	.0245	.0493	.0245	.0493	.0244

TABLE B-V. – CONTINUED

(i) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $80 \leqq XX \leqq 89$

XX / $\frac{x_n-x_o}{\Delta x}$	80		81		82		83		84		85		86		87		88		89	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
0.XX	0.8109	0.3513	0.8041	0.3487	0.7973	0.3462	0.7906	0.3438	0.7841	0.3413	0.7777	0.3389	0.7714	0.3366	0.7652	0.3343	0.7591	0.3320	0.7531	0.3297
1.	.4418	.2047	.4399	.2039	.4379	.2030	.4360	.2022	.4340	.2014	.4321	.2006	.4302	.1998	.4284	.1989	.4265	.1981	.4247	.1974
2.	.3054	.1449	.3044	.1445	.3035	.1441	.3026	.1437	.3017	.1433	.3008	.1428	.2998	.1424	.2989	.1420	.2980	.1416	.2972	.1412
3.	.2336	.1123	.2331	.1120	.2325	.1118	.2320	.1115	.2314	.1113	.2309	.1110	.2304	.1108	.2298	.1105	.2293	.1103	.2288	.1100
4.	.1892	.0917	.1889	.0915	.1885	.0913	.1882	.0911	.1878	.0910	.1875	.0908	.1871	.0906	.1868	.0905	.1864	.0903	.1861	.0902
5.	.1591	.0774	.1588	.0773	.1586	.0772	.1583	.0770	.1581	.0770	.1578	.0768	.1576	.0767	.1573	.0766	.1571	.0765	.1568	.0764
6.	.1372	.0670	.1370	.0669	.1368	.0668	.1366	.0668	.1365	.0667	.1363	.0666	.1361	.0665	.1359	.0664	.1357	.0663	.1355	.0662
7.	.1206	.0591	.1205	.0590	.1203	.0589	.1202	.0589	.1200	.0588	.1199	.0587	.1198	.0587	.1196	.0587	.1195	.0586	.1193	.0585
8	.1076	.0528	.1075	.0528	.1074	.0527	.1073	.0527	.1072	.0526	.1071	.0526	.1069	.0525	.1068	.0525	.1067	.0524	.1066	.0523
9.	.0972	.0478	.0971	.0477	.0970	.0477	.0969	.0477	.0968	.0476	.0967	.0476	.0966	.0475	.0965	.0475	.0964	.0475	.0963	.0474
10.	.0886	.0436	.0885	.0436	.0884	.0436	.0883	.0435	.0882	.0435	.0882	.0434	.0881	.0434	.0880	.0434	.0879	.0433	.0879	.0433
11.	.0814	.0401	.0813	.0401	.0812	.0401	.0811	.0401	.0811	.0401	.0810	.0400	.0810	.0399	.0809	.0399	.0808	.0398	.0808	.0398
12.	.0752	.0371	.0752	.0371	.0751	.0371	.0751	.0371	.0750	.0370	.0749	.0370	.0749	.0370	.0748	.0369	.0748	.0369	.0747	.0369
13.	.0700	.0346	.0699	.0345	.0699	.0345	.0698	.0345	.0698	.0345	.0697	.0345	.0697	.0344	.0696	.0344	.0696	.0344	.0695	.0344
14.	.0654	.0323	.0653	.0323	.0653	.0323	.0653	.0323	.0652	.0323	.0652	.0322	.0651	.0322	.0651	.0322	.0650	.0322	.0650	.0322
15.	.0614	.0304	.0613	.0303	.0613	.0303	.0613	.0303	.0612	.0303	.0612	.0303	.0611	.0303	.0611	.0302	.0611	.0302	.0610	.0302
16.	.0578	.0286	.0578	.0286	.0578	.0286	.0577	.0286	.0577	.0286	.0577	.0285	.0576	.0285	.0576	.0285	.0576	.0285	.0575	.0285
17.	.0547	.0271	.0546	.0271	.0546	.0270	.0546	.0270	.0545	.0270	.0545	.0270	.0545	.0270	.0545	.0270	.0544	.0270	.0544	.0270
18.	.0518	.0257	.0518	.0257	.0518	.0257	.0517	.0257	.0517	.0256	.0517	.0256	.0517	.0256	.0516	.0256	.0516	.0256	.0516	.0256
19.XX	.0493	.0244	.0492	.0244	.0492	.0244	.0492	.0244	.0492	.0244	.0491	.0244	.0491	.0244	.0491	.0244	.0491	.0243	.0491	.0243

TABLE B-V.– CONCLUDED

(j) $0.XX \leqq \frac{x_n - x_o}{\Delta x} \leqq 19.XX$ where $90 \leqq XX \leqq 99$

XX / $\frac{x_n-x_o}{\Delta x}$	90		91		92		93		94		95		96		97		98		99	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
0.XX	0.7472	0.3275	0.7414	0.3253	0.7357	0.3231	0.7301	0.3210	0.7245	0.3189	0.7191	0.3168	0.7138	0.3148	0.7085	0.3128	0.7033	0.3108	0.6982	0.3088
1.	.4229	.1966	.4211	.1958	.4193	.1950	.4175	.1943	.4157	.1935	.4140	.1927	.4122	.1920	.4105	.1913	.4088	.1905	.4071	.1898
2.	.2963	.1408	.2954	.1404	.2945	.1400	.2936	.1396	.2928	.1392	.2919	.1389	.2911	.1385	.2902	.1381	.2894	.1377	.2885	.1373
3.	.2283	.1098	.2277	.1096	.2272	.1093	.2267	.1091	.2262	.1088	.2257	.1086	.2252	.1083	.2247	.1081	.2242	.1079	.2236	.1076
4.	.1857	.0900	.1854	.0898	.1850	.0897	.1847	.0895	.1843	.0893	.1840	.0892	.1837	.0890	.1833	.0889	.1830	.0887	.1827	.0886
5.	.1566	.0762	.1563	.0761	.1561	.0760	.1558	.0759	.1556	.0758	.1554	.0757	.1551	.0756	.1549	.0755	.1546	.0753	.1544	.0752
6.	.1353	.0661	.1352	.0660	.1350	.0660	.1348	.0659	.1346	.0658	.1344	.0657	.1343	.0656	.1341	.0656	.1339	.0655	.1337	.0654
7.	.1192	.0584	.1190	.0584	.1189	.0583	.1188	.0582	.1186	.0582	.1185	.0580	.1183	.0580	.1182	.0579	.1181	.0579	.1179	.0578
8.	.1065	.0523	.1064	.0523	.1063	.0522	.1061	.0521	.1060	.0520	.1059	.0520	.1058	.0519	.1057	.0519	.1056	.0518	.1055	.0518
9.	.0962	.0473	.0961	.0473	.0960	.0473	.0960	.0472	.0959	.0472	.0958	.0471	.0957	.0471	.0956	.0470	.0955	.0470	.0954	.0469
10.	.0878	.0433	.0877	.0432	.0876	.0432	.0875	.0432	.0875	.0431	.0874	.0431	.0873	.0431	.0872	.0430	.0872	.0430	.0871	.0429
11.	.0807	.0398	.0806	.0398	.0806	.0398	.0805	.0397	.0804	.0397	.0804	.0397	.0803	.0396	.0802	.0396	.0802	.0396	.0801	.0396
12.	.0747	.0369	.0746	.0368	.0746	.0368	.0745	.0368	.0744	.0368	.0744	.0367	.0743	.0367	.0743	.0367	.0742	.0367	.0742	.0366
13.	.0695	.0343	.0694	.0343	.0694	.0343	.0693	.0343	.0693	.0342	.0692	.0342	.0692	.0342	.0691	.0342	.0691	.0342	.0690	.0341
14.	.0650	.0321	.0649	.0321	.0649	.0321	.0648	.0321	.0648	.0321	.0647	.0320	.0647	.0320	.0647	.0320	.0646	.0320	.0646	.0319
15.	.0610	.0302	.0610	.0302	.0609	.0302	.0609	.0301	.0608	.0301	.0608	.0301	.0608	.0301	.0607	.0301	.0607	.0300	.0607	.0300
16.	.0575	.0285	.0575	.0285	.0574	.0284	.0574	.0284	.0574	.0284	.0573	.0284	.0573	.0284	.0573	.0284	.0572	.0283	.0572	.0283
17.	.0544	.0269	.0543	.0269	.0543	.0269	.0543	.0269	.0542	.0269	.0542	.0269	.0542	.0269	.0542	.0268	.0541	.0268	.0541	.0268
18.	.0516	.0256	.0515	.0255	.0515	.0255	.0515	.0255	.0515	.0255	.0514	.0255	.0514	.0255	.0514	.0255	.0513	.0254	.0513	.0254
19.XX	.0490	.0243	.0490	.0243	.0490	.0243	.0490	.0243	.0489	.0243	.0489	.0243	.0489	.0243	.0489	.0242	.0488	.0242	.0488	.0242

TABLE B-VI.– VALUES OF j_{no} AND j_{no}^* USED IN EVALUATING EQUATION (26)

$$20.0 \leqq \frac{x_n - x_o}{\Delta x} \leqq 39.9$$

X → / $\frac{x_n-x_o}{\Delta x}$ ↓	0		1		2		3		4		5		6		7		8		9	
	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
20.X	0.0488	0.0242	0.0486	0.0241	0.0483	0.0241	0.0481	0.0239	0.0478	0.0237	0.0476	0.0236	0.0474	0.0236	0.0472	0.0234	0.0470	0.0232	0.0467	0.0231
21.	.0465	.0231	.0463	.0230	.0461	.0229	.0459	.0227	.0457	.0226	.0455	.0226	.0453	.0224	.0451	.0224	.0448	.0223	.0446	.0222
22.	.0444	.0221	.0443	.0220	.0441	.0219	.0439	.0218	.0437	.0217	.0435	.0216	.0433	.0215	.0431	.0214	.0429	.0213	.0427	.0212
23.	.0426	.0211	.0424	.0210	.0422	.0210	.0420	.0209	.0418	.0208	.0417	.0207	.0415	.0206	.0413	.0205	.0412	.0204	.0410	.0204
24.	.0408	.0203	.0407	.0202	.0405	.0201	.0403	.0200	.0402	.0199	.0400	.0199	.0398	.0198	.0397	.0197	.0395	.0196	.0394	.0195
25.	.0392	.0195	.0391	.0194	.0389	.0193	.0388	.0192	.0386	.0192	.0385	.0191	.0383	.0190	.0382	.0190	.0380	.0189	.0379	.0188
26.	.0377	.0187	.0376	.0187	.0375	.0186	.0373	.0185	.0372	.0185	.0370	.0184	.0369	.0184	.0368	.0183	.0366	.0182	.0365	.0182
27.	.0364	.0181	.0362	.0180	.0361	.0179	.0360	.0179	.0358	.0178	.0357	.0178	.0356	.0177	.0355	.0176	.0353	.0176	.0352	.0175
28.	.0351	.0175	.0350	.0174	.0348	.0173	.0347	.0173	.0346	.0172	.0345	.0172	.0344	.0171	.0342	.0171	.0341	.0170	.0340	.0169
29.	.0339	.0168	.0338	.0168	.0337	.0167	.0336	.0167	.0334	.0166	.0333	.0166	.0332	.0165	.0331	.0165	.0330	.0164	.0329	.0164
30.	.0328	.0163	.0327	.0162	.0326	.0162	.0325	.0161	.0324	.0161	.0323	.0160	.0322	.0160	.0321	.0159	.0320	.0158	.0319	.0158
31.	.0317	.0158	.0316	.0157	.0315	.0157	.0315	.0156	.0314	.0156	.0313	.0155	.0312	.0155	.0311	.0154	.0310	.0154	.0309	.0153
32.	.0308	.0153	.0307	.0152	.0306	.0152	.0305	.0151	.0304	.0151	.0303	.0151	.0302	.0150	.0301	.0150	.0300	.0150	.0299	.0149
33.	.0299	.0149	.0298	.0148	.0297	.0148	.0296	.0147	.0295	.0147	.0294	.0146	.0293	.0146	.0292	.0145	.0292	.0145	.0291	.0145
34.	.0290	.0144	.0289	.0144	.0288	.0143	.0287	.0143	.0287	.0143	.0286	.0142	.0285	.0142	.0284	.0142	.0283	.0141	.0283	.0141
35.	.0282	.0141	.0281	.0140	.0280	.0139	.0279	.0139	.0279	.0138	.0278	.0138	.0277	.0138	.0276	.0138	.0276	.0137	.0275	.0137
36.	.0274	.0136	.0273	.0136	.0273	.0136	.0272	.0135	.0271	.0135	.0270	.0134	.0270	.0134	.0269	.0134	.0268	.0133	.0267	.0133
37.	.0267	.0133	.0266	.0132	.0265	.0132	.0265	.0132	.0264	.0131	.0263	.0131	.0262	.0130	.0262	.0130	.0261	.0130	.0260	.0130
38.	.0260	.0129	.0259	.0129	.0258	.0129	.0258	.0128	.0257	.0128	.0256	.0127	.0256	.0127	.0255	.0127	.0254	.0127	.0254	.0126
39.X	.0253	.0126	.0253	.0126	.0252	.0126	.0251	.0125	.0251	.0125	.0250	.0125	.0249	.0124	.0249	.0124	.0248	.0124	.0248	.0123

TABLE B-VII.— VALUES OF j_{no} AND j_{no}^* USED IN EVALUATING EQUATION (26)

$$40.0 \leqq \frac{x_n - x_o}{\Delta x} \leqq 89.5$$

$\frac{x_n-x_o}{\Delta x}$	4X.0		4X.5		5X.0		5X.5		6X.0		6X.5		7X.0		7X.5		8X.0		8X.5	
X	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
0	0.0247	0.0123	0.0244	0.0122	0.0198	0.0099	0.0196	0.0097	0.0165	0.0082	0.0164	0.0082	0.0142	0.0071	0.0141	0.0071	0.0124	0.0062	0.0123	0.0061
1	.0241	.0120	.0238	.0118	.0194	.0097	.0192	.0096	.0163	.0081	.0161	.0081	.0140	.0069	.0139	.0069	.0123	.0061	.0122	.0061
2	.0235	.0117	.0233	.0116	.0190	.0095	.0189	.0094	.0160	.0080	.0159	.0079	.0138	.0068	.0137	.0068	.0121	.0061	.0120	.0060
3	.0230	.0114	.0227	.0113	.0187	.0093	.0185	.0092	.0157	.0079	.0156	.0078	.0136	.0068	.0135	.0068	.0120	.0059	.0119	.0059
4	.0225	.0112	.0222	.0110	.0184	.0091	.0182	.0090	.0155	.0077	.0154	.0077	.0134	.0066	.0133	.0066	.0118	.0059	.0118	.0059
5	.0220	.0109	.0217	.0108	.0180	.0090	.0179	.0089	.0153	.0076	.0152	.0075	.0132	.0066	.0132	.0065	.0117	.0058	.0116	.0058
6	.0215	.0107	.0213	.0106	.0177	.0089	.0175	.0088	.0150	.0075	.0149	.0074	.0131	.0065	.0130	.0065	.0116	.0058	.0115	.0058
7	.0211	.0105	.0208	.0103	.0174	.0087	.0172	.0086	.0148	.0074	.0147	.0073	.0129	.0064	.0128	.0064	.0114	.0057	.0114	.0056
8	.0206	.0103	.0204	.0101	.0171	.0085	.0169	.0085	.0146	.0073	.0145	.0072	.0127	.0063	.0127	.0063	.0113	.0056	.0112	.0056
9	.0202	.0101	.0200	.0100	.0168	.0084	.0167	.0083	.0144	.0072	.0143	.0072	.0126	.0063	.0125	.0062	.0112	.0055	.0111	.0055

TABLE B-VIII.— VALUES OF j_{no} AND j_{no}^* USED IN EVALUATING EQUATION (26)

$$90 \leqq \frac{x_n - x_o}{\Delta x} \leqq 189$$

$\frac{x_n-x_o}{\Delta x}$	9X.0		10X.0		11X.0		12X.0		13X.0		14X.0		15X.0		16X.0		17X.0		18X.0	
X	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
0	0.0111	0.0055	0.0100	0.0050	0.0091	0.0045	0.0083	0.0042	0.0077	0.0038	0.0071	0.0036	0.0067	0.0033	0.0063	0.0031	0.0059	0.0029	0.0056	0.0028
1	.0109	.0054	.0099	.0050	.0090	.0045	.0083	.0041	.0076	.0038	.0071	.0035	.0066	.0033	.0062	.0031	.0058	.0029	.0055	.0028
2	.0108	.0054	.0098	.0049	.0089	.0045	.0082	.0041	.0076	.0038	.0070	.0035	.0066	.0033	.0062	.0031	.0058	.0029	.0055	.0027
3	.0107	.0054	.0097	.0049	.0088	.0044	.0081	.0041	.0075	.0038	.0070	.0035	.0065	.0033	.0061	.0031	.0058	.0029	.0055	.0027
4	.0106	.0053	.0096	.0048	.0088	.0044	.0081	.0040	.0074	.0037	.0069	.0035	.0065	.0032	.0061	.0031	.0057	.0029	.0054	.0027
5	.0105	.0053	.0095	.0048	.0087	.0043	.0080	.0040	.0074	.0037	.0069	.0034	.0065	.0032	.0061	.0030	.0057	.0029	.0054	.0027
6	.0104	.0052	.0094	.0047	.0086	.0043	.0079	.0040	.0074	.0037	.0068	.0034	.0064	.0032	.0060	.0030	.0057	.0028	.0054	.0027
7	.0103	.0052	.0093	.0047	.0085	.0043	.0079	.0039	.0073	.0036	.0068	.0034	.0064	.0032	.0060	.0030	.0056	.0028	.0053	.0027
8	.0102	.0051	.0093	.0046	.0085	.0042	.0078	.0039	.0072	.0036	.0068	.0034	.0063	.0032	.0060	.0030	.0056	.0028	.0053	.0027
9	.0101	.0051	.0092	.0046	.0084	.0042	.0078	.0039	.0072	.0036	.0067	.0034	.0063	.0031	.0059	.0030	.0056	.0028	.0053	.0026

TABLE B-IX.– VALUES OF j_{no} AND j_{no}^* USED IN EVALUATING EQUATION (26)

(a) $-0.999 \leq \frac{x_n - x_o}{\Delta x} \leq -0.750$

X / $\frac{x_n-x_o}{\Delta x}$	9		8		7		6		5		4		3		2		1		0	
	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
−.99X	−6.9068	−5.8998	−6.2126	−5.2002	−5.8061	−4.7887	−5.5175	−4.4954	−5.2933	−4.2668	−5.1100	−4.0793	−4.9548	−3.9202	−4.8203	−3.7817	−4.7015	−3.6592	−4.5951	−3.5492
−.98	−4.4988	−3.4493	−4.4108	−3.3578	−4.3297	−3.2734	−4.2546	−3.1753	−4.1846	−3.1218	−4.1190	−3.0531	−4.0574	−2.9884	−3.9992	−2.9272	−3.9441	−2.8692	−3.8918	−2.8140
−.97	−3.8420	−2.7613	−3.7945	−2.7110	−3.7489	−2.6627	−3.7054	−2.6165	−3.6636	−2.5720	−3.6233	−2.5291	−3.5845	−2.4878	−3.5473	−2.4480	−3.5110	−2.4092	−3.4761	−2.3718
−.96	−3.4423	−2.3356	−3.4095	−2.3004	−3.3777	−2.2662	−3.3468	−2.2330	−3.3168	−2.2007	−3.2876	−2.1692	−3.2591	−2.1385	−3.2314	−2.1086	−3.2044	−2.0794	−3.1781	−2.0509
−.95	−3.1523	−2.0231	−3.1272	−1.9959	−3.1026	−1.9692	−3.0786	−1.9431	−3.0550	−1.9176	−3.0320	−1.8925	−3.0095	−1.8680	−2.9873	−1.8440	−2.9657	−1.8204	−2.9444	−1.7972
−.94	−2.9236	−1.7745	−2.9031	−1.7522	−2.8830	−1.7302	−2.8632	−1.7086	−2.8439	−1.6875	−2.8248	−1.6666	−2.8060	−1.6461	−2.7875	−1.6259	−2.7694	−1.6060	−2.7516	−1.5865
−.93	−2.7339	−1.5671	−2.7167	−1.5482	−2.6995	−1.5295	−2.6827	−1.5110	−2.6662	−1.4729	−2.6499	−1.4750	−2.6337	−1.4572	−2.6178	−1.4398	−2.6022	−1.4226	−2.5867	−1.4056
−.92	−2.5715	−1.3889	−2.5564	−1.3723	−2.5415	−1.3560	−2.5268	−1.3399	−2.5123	−1.3239	−2.4980	−1.3081	−2.4838	−1.2926	−2.4699	−1.2772	−2.4560	−1.2620	−2.4423	−1.2470
−.91	−2.4289	−1.2321	−2.4155	−1.2174	−2.4022	−1.2029	−2.3892	−1.1885	−2.3763	−1.1743	−2.3635	−1.1602	−2.3508	−1.1463	−2.3383	−1.1326	−2.3259	−1.1189	−2.3136	−1.1054
−.90	−2.3015	−1.0920	−2.2895	−1.0788	−2.2775	−1.0657	−2.2657	−1.0528	−2.2541	−1.0399	−2.2425	−1.0272	−2.2310	−1.0146	−2.2196	−1.0021	−2.2084	−.9898	−2.1972	−.9775
−.89	−2.1862	−.9654	−2.1752	−.9533	−2.1643	−.9414	−2.1536	−.9296	−2.1429	−.9179	−2.1323	−.9063	−2.1218	−.8947	−2.1113	−.8833	−2.1010	−.8720	−2.0908	−.8608
−.88	−2.0806	−.8496	−2.0705	−.8386	−2.0605	−.8276	−2.0505	−.8168	−2.0407	−.8060	−2.0309	−.7953	−2.0212	−.7847	−2.0115	−.7742	−2.0019	−.7637	−1.9924	−.7533
−.87	−1.9830	−.7431	−1.9736	−.7328	−1.9643	−.7227	−1.9551	−.7127	−1.9459	−.7027	−1.9368	−.6928	−1.9277	−.6829	−1.9188	−.6732	−1.9098	−.6635	−1.9010	−.6538
−.86	−1.8921	−.6443	−1.8834	−.6348	−1.8747	−.6254	−1.8660	−.6160	−1.8575	−.6067	−1.8489	−.5975	−1.8404	−.5883	−1.8320	−.5792	−1.8236	−.5701	−1.8153	−.5612
−.85	−1.8070	−.5522	−1.7988	−.5434	−1.7906	−.5345	−1.7825	−.5258	−1.7744	−.5171	−1.7663	−.5084	−1.7583	−.4999	−1.7504	−.4913	−1.7425	−.4828	−1.7346	−.4744
−.84	−1.7268	−.4660	−1.7190	−.4577	−1.7113	−.4495	−1.7036	−.4412	−1.6959	−.4330	−1.6883	−.4249	−1.6807	−.4168	−1.6732	−.4088	−1.6657	−.4008	−1.6582	−.3929
−.83	−1.6508	−.3850	−1.6434	−.3772	−1.6361	−.3694	−1.6288	−.3617	−1.6215	−.3539	−1.6142	−.3463	−1.6070	−.3387	−1.5999	−.3311	−1.5927	−.3244	−1.5856	−.3161
−.82	−1.5786	−.3086	−1.5715	−.3012	−1.5645	−.2938	−1.5575	−.2865	−1.5506	−.2792	−1.5437	−.2720	−1.5368	−.2648	−1.5300	−.2576	−1.5231	−.2505	−1.5164	−.2434
−.81	−1.5096	−.2364	−1.5029	−.2293	−1.4962	−.2224	−1.4895	−.2154	−1.4828	−.2085	−1.4762	−.2016	−1.4696	−.1948	−1.4630	−.1880	−1.4565	−.1812	−1.4500	−.1745
−.80	−1.4435	−.1678	−1.4371	−.1611	−1.4306	−.1545	−1.4242	−.1479	−1.4178	−.1414	−1.4115	−.1348	−1.4051	−.1283	−1.3988	−.1219	−1.3926	−.1154	−1.3863	−.1090
−.79	−1.3801	−.1027	−1.3738	−.0963	−1.3676	−.0900	−1.3615	−.0837	−1.3553	−.0775	−1.3492	−.0713	−1.3431	−.0651	−1.3370	−.0589	−1.3310	−.0528	−1.3249	−.0467
−.78	−1.3189	−.0406	−1.3129	−.0346	−1.3069	−.0286	−1.3010	−.0226	−1.2951	−.0166	−1.2891	−.0107	−1.2832	−.0048	−1.2774	.0011	−1.2715	.0070	−1.2657	.0128
−.77	−1.2599	.0186	−1.2540	.0244	−1.2483	.0301	−1.2425	.0358	−1.2368	.0415	−1.2310	.0472	−1.2253	.0528	−1.2196	.0584	−1.2140	.0640	−1.2083	.0696
−.76	−1.2027	.0751	−1.1970	.0807	−1.1914	.0862	−1.1859	.0916	−1.1803	.0971	−1.1747	.1025	−1.1692	.1079	−1.1637	.1133	−1.1582	.1186	−1.1527	.1240
−.75X	−1.1472	.1293	−1.1417	.1346	−1.1363	.1398	−1.1309	.1451	−1.1254	.1503	−1.1200	.1555	−1.1147	.1606	−1.1093	.1658	−1.1040	.1709	−1.0986	.1760

TABLE B-IX.– CONTINUED

(b) $-0.749 \leq \frac{x_n - x_o}{\Delta x} \leq -0.500$

$\frac{x_n - x_o}{\Delta x}$ \ X	9		8		7		6		5	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
−.74X	−1.0933	0.1811	−1.0880	0.1862	−1.0827	0.1912	−1.0774	0.1963	−1.0721	0.2013
−.73	−1.0408	.2309	−1.0356	.2357	−1.0304	.2406	−1.0253	.2454	−1.0201	.2502
−.72	−.9895	.2786	−.9845	.2833	−.9795	.2879	−.9744	.2926	−.9694	.2972
−.71	−.9395	.3245	−.9346	.3290	−.9296	.3334	−.9247	.3379	−.9198	.3423
−.70	−.8905	.3686	−.8857	.3729	−.8809	.3772	−.8761	.3815	−.8712	.3858
−.69	−.8426	.4111	−.8378	.4152	−.8330	.4194	−.8283	.4235	−.8236	.4276
−.68	−.7954	.4519	−.7908	.4559	−.7861	.4599	−.7815	.4639	−.7768	.4679
−.67	−.7492	.4913	−.7446	.4952	−.7400	.4990	−.7354	.5028	−.7309	.5067
−.66	−.7036	.5293	−.6991	.5330	−.6946	.5367	−.6901	.5404	−.6857	.5440
−.65	−.6589	.5658	−.6544	.5694	−.6500	.5730	−.6455	.5765	−.6411	.5801
−.64	−.6146	.6011	−.6103	.6046	−.6059	.6080	−.6015	.6114	−.5971	.6149
−.63	−.5710	.6351	−.5667	.6385	−.5624	.6418	−.5581	.6451	−.5537	.6484
−.62	−.5279	.6679	−.5237	.6711	−.5194	.6743	−.5151	.6776	−.5108	.6807
−.61	−.4853	.6996	−.4811	.7027	−.4769	.7058	−.4726	.7089	−.4684	.7119
−.60	−.4431	.7302	−.4389	.7331	−.4347	.7361	−.4305	.7391	−.4264	.7420
−.59	−.4013	.7596	−.3972	.7625	−.3930	.7654	−.3888	.7683	−.3847	.7711
−.58	−.3598	.7881	−.3557	.7908	−.3516	.7936	−.3475	.7964	−.3433	.7991
−.57	−.3187	.8155	−.3146	.8182	−.3105	.8209	−.3064	.8235	−.3023	.8262
−.56	−.2778	.8419	−.2737	.8449	−.2696	.8471	−.2656	.8497	−.2604	.8529
−.55	−.2371	.8675	−.2331	.8700	−.2290	.8724	−.2249	.8749	−.2209	.8774
−.54	−.1966	.8920	−.1926	.8945	−.1886	.8969	−.1845	.8993	−.1805	.9016
−.53	−.1563	.9157	−.1523	.9181	−.1483	.9204	−.1443	.9227	−.1402	.9250
−.52	−.1161	.9386	−.1121	.9408	−.1081	.9430	−.1041	.9452	−.1001	.9475
−.51	−.0760	.9605	−.0720	.9627	−.0680	.9648	−.0640	.9670	−.0600	.9691
−.50X	−.0360	.9817	−.0320	.9837	−.0280	.9858	−.0240	.9879	−.0200	.9899

$\frac{x_n - x_o}{\Delta x}$ \ X	4		3		2		1		0	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
−.74X	−1.0669	0.2062	−1.0616	0.2112	−1.0564	0.2162	−1.0512	0.2211	−1.0460	0.2260
−.73	−1.0150	.2550	−1.0099	.2598	−1.0048	.2645	−.9997	.2692	−.9946	.2739
−.72	−.9644	.3018	−.9594	.3064	−.9544	.3109	−.9494	.3155	−.9445	.3200
−.71	−.9149	.3468	−.9100	.3512	−.9051	.3556	−.9002	.3599	−.8954	.3643
−.70	−.8664	.3900	−.8616	.3943	−.8568	.3985	−.8521	.4020	−.8473	.4069
−.69	−.8189	.4317	−.8142	.4358	−.8095	.4398	−.8048	.4439	−.8001	.4479
−.68	−.7722	.4718	−.7676	.4757	−.7630	.4796	−.7584	.4835	−.7538	.4874
−.67	−.7263	.5104	−.7218	.5142	−.7173	.5180	−.7127	.5218	−.7082	.5255
−.66	−.6812	.5477	−.6767	.5513	−.6722	.5550	−.6678	.5586	−.6633	.5622
−.65	−.6367	.5836	−.6322	.5872	−.6278	.5914	−.6234	.5922	−.6191	.5976
−.64	−.5928	.6183	−.5884	.6217	−.5841	.6250	−.5797	.6284	−.5754	.6318
−.63	−.5494	.6517	−.5451	.6549	−.5408	.6582	−.5365	.6615	−.5322	.6647
−.62	−.5066	.6839	−.5023	.6871	−.4980	.6902	−.4938	.6934	−.4896	.6965
−.61	−.4642	.7150	−.4600	.7180	−.4557	.7211	−.4515	.7241	−.4473	.7271
−.60	−.4222	.7450	−.4180	.7480	−.4138	.7509	−.4097	.7538	−.4055	.7567
−.59	−.3806	.7739	−.3764	.7768	−.3723	.7796	−.3680	.7824	−.3639	.7853
−.58	−.3392	.8019	−.3351	.8046	−.3310	.8074	−.3269	.8101	−.3228	.8128
−.57	−.2982	.8288	−.2941	.8315	−.2900	.8341	−.2859	.8367	−.2818	.8393
−.56	−.2574	.8548	−.2533	.8574	−.2493	.8599	−.2452	.8624	−.2412	.8649
−.55	−.2169	.8799	−.2128	.8823	−.2088	.8848	−.2047	.8873	−.2007	.8896
−.54	−.1765	.9040	−.1724	.9064	−.1684	.9087	−.1644	.9111	−.1603	.9134
−.53	−.1362	.9273	−.1322	.9295	−.1282	.9318	−.1242	.9341	−.1201	.9363
−.52	−.0961	.9497	−.0921	.9519	−.0881	.9540	−.0840	.9562	−.0800	.9584
−.51	−.0560	.9712	−.0520	.9733	−.0480	.9754	−.0440	.9775	−.0400	.9796
−.50X	−.0160	.9919	−.0120	.9940	−.0080	.9960	−.0040	.9980	0	1.0000

TABLE B-IX.– CONTINUED

(c) $-0.499 \leqq \frac{x_n - x_o}{\Delta x} \leqq -0.250$

I	9		8		7		6		5		4		3		2		1		0	
$\frac{x_n-x_o}{\Delta x}$	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*	j_{no}	j_{no}^*
–.49X	0.0040	1.0020	0.0080	1.0040	0.0120	1.0060	0.0160	1.0079	0.0200	1.0099	0.0240	1.0119	0.0280	1.0138	0.0320	1.0157	0.0360	1.0177	0.0400	1.0196
–.48	.0440	1.0215	.0480	1.0234	.0520	1.0253	.0560	1.0272	.0601	1.0291	.0640	1.0310	.0680	1.0329	.0720	1.0347	.0760	1.0366	.0800	1.0384
–.47	.0841	1.0403	.0881	1.0421	.0920	1.0439	.0960	1.0457	.1001	1.0476	.1041	1.0493	.1081	1.0511	.1121	1.0529	.1161	1.0547	.1202	1.0565
–.46	.1243	1.0582	.1282	1.0600	.1322	1.0617	.1362	1.0635	.1402	1.0652	.1443	1.0669	.1482	1.0686	.1523	1.0704	.1563	1.0721	.1603	1.0738
–.45	.1643	1.0754	.1684	1.0771	.1724	1.0788	.1765	1.0805	.1805	1.0821	.1845	1.0838	.1886	1.0854	.1926	1.0871	.1966	1.0887	.2007	1.0903
–.44	.2047	1.0919	.2087	1.0935	.2128	1.0951	.2169	1.0967	.2209	1.0983	.2250	1.0999	.2290	1.1014	.2330	1.1030	.2371	1.1046	.2411	1.1061
–.43	.2452	1.1077	.2493	1.1092	.2533	1.1107	.2574	1.1122	.2615	1.1138	.2655	1.1152	.2696	1.1168	.2737	1.1182	.2778	1.1197	.2819	1.1212
–.42	.2859	1.1227	.2900	1.1241	.2941	1.1256	.2982	1.1270	.3023	1.1285	.3064	1.1299	.3105	1.1313	.3146	1.1328	.3187	1.1342	.3227	1.1355
–.41	.3269	1.1370	.3310	1.1383	.3351	1.1397	.3392	1.1411	.3433	1.1425	.3475	1.1439	.3516	1.1452	.3557	1.1466	.3598	1.1479	.3639	1.1492
–.40	.3681	1.1506	.3723	1.1519	.3764	1.1532	.3806	1.1545	.3847	1.1558	.3888	1.1571	.3930	1.1584	.3972	1.1597	.4013	1.1609	.4055	1.1622
–.39	.4097	1.1635	.4138	1.1647	.4180	1.1659	.4222	1.1672	.4264	1.6842	.4305	1.1696	.4347	1.1708	.4389	1.1720	.4431	1.1732	.4473	1.1745
–.38	.4515	1.1756	.4557	1.1768	.4600	1.1780	.4642	1.1792	.4684	1.1803	.4726	1.1815	.4769	1.1826	.4811	1.1838	.4853	1.1849	.4896	1.1860
–.37	.4938	1.1871	.4980	1.1883	.5023	1.1894	.5066	1.1905	.5108	1.1916	.5151	1.1926	.5194	1.1937	.5237	1.1948	.5279	1.1959	.5322	1.1969
–.36	.5365	1.1980	.5408	1.1990	.5451	1.2001	.5494	1.2011	.5537	1.2021	.5581	1.2031	.5624	1.2041	.5667	1.2051	.5710	1.2061	.5754	1.2071
–.35	.5797	1.2081	.5841	1.2091	.5884	1.2101	.5927	1.2110	.5971	1.2120	.6015	1.2129	.6059	1.2139	.6103	1.2148	.6146	1.2157	.6191	1.2167
–.34	.6234	1.2176	.6278	1.2185	.6322	1.2194	.6367	1.2203	.6411	1.2212	.6455	1.2221	.6500	1.2229	.6544	1.2238	.6589	1.2247	.6633	1.2255
–.33	.6678	1.2264	.6722	1.2272	.6767	1.2281	.6812	1.2289	.6857	1.2297	.6901	1.2305	.6946	1.2313	.6991	1.2321	.7036	1.2329	.7082	1.2337
–.32	.7127	1.2345	.7173	1.2353	.7218	1.2360	.7263	1.2368	.7309	1.2375	.7354	1.2383	.7400	1.2390	.7446	1.2398	.7492	1.2405	.7538	1.2412
–.31	.7584	1.2419	.7630	1.2426	.7676	1.2433	.7722	1.2440	.7768	1.2447	.7815	1.2454	.7861	1.2461	.7908	1.2467	.7954	1.2474	.8001	1.2480
–.30	.8048	1.2487	.8095	1.2493	.8142	1.2500	.8189	1.2506	.8236	1.2512	.8283	1.2518	.8330	1.2524	.8378	1.2530	.8426	1.2536	.8473	1.2542
–.29	.8521	1.2548	.8568	1.2553	.8616	1.2559	.8664	1.2565	.8712	1.2570	.8761	1.2576	.8809	1.2581	.8857	1.2586	.8905	1.2591	.8954	1.2597
–.28	.9002	1.2602	.9051	1.2607	.9100	1.2612	.9149	1.2617	.9198	1.2624	.9247	1.2626	.9296	1.2631	.9346	1.2635	.9395	1.2640	.9445	1.2644
–.27	.9494	1.2649	.9544	1.2653	.9594	1.2658	.9644	1.2662	.9694	1.2666	.9744	1.2670	.9795	1.2674	.9845	1.2678	.9895	1.2682	.9946	1.2685
–.26	.9997	1.2689	1.0048	1.2693	1.0099	1.2696	1.0150	1.2700	1.0201	1.2703	1.0253	1.2707	1.0304	1.2710	1.0356	1.2713	1.0408	1.2716	1.0460	1.2720
–.25X	1.0512	1.2723	1.0564	1.2726	1.0616	1.2728	1.0669	1.2731	1.0721	1.2734	1.0774	1.2737	1.0827	1.2739	1.0880	1.2742	1.0933	1.2744	1.0986	1.2747

TABLE B-IX.– CONCLUDED

(d) $-0.249 \leq \frac{x_n - x_o}{\Delta x} \leq 0.000$

$\frac{x_n-x_o}{\Delta x}$ \ X	9		8		7		6		5		4		3		2		1		0	
	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*	J_{no}	J_{no}^*
–.24X	1.1040	1.2749	1.1093	1.2751	1.1147	1.2753	1.1200	1.2755	1.1254	1.2757	1.1309	1.2759	1.1363	1.2761	1.1417	1.2763	1.1472	1.2765	1.1527	1.2766
–.23	1.1582	1.2768	1.1637	1.2770	1.1692	1.2771	1.1747	1.2772	1.1803	1.2774	1.1859	1.2775	1.1914	1.2776	1.1970	1.2777	1.2027	1.2778	1.2083	1.2779
–.22	1.2140	1.2780	1.2196	1.2781	1.2253	1.2782	1.2310	1.2782	1.2368	1.2783	1.2425	1.2783	1.2483	1.2784	1.2540	1.2784	1.2599	1.2784	1.2657	1.2784
–.21	1.2715	1.2785	1.2774	1.2785	1.2832	1.2785	1.2891	1.2785	1.2951	1.2784	1.3010	1.2784	1.3069	1.2784	1.3129	1.2783	1.3189	1.2783	1.3249	1.2782
–.20	1.3310	1.2782	1.3370	1.2781	1.3431	1.2780	1.3492	1.2779	1.3553	1.2778	1.3615	1.2777	1.3676	1.2776	1.3738	1.2775	1.3801	1.2774	1.3863	1.2773
–.19	1.3926	1.2771	1.3988	1.2770	1.4051	1.2768	1.4115	1.2766	1.4178	1.2765	1.4242	1.2763	1.4306	1.2761	1.4371	1.2759	1.4435	1.2757	1.4500	1.2755
–.18	1.4565	1.2753	1.4630	1.2751	1.4696	1.2748	1.4762	1.2746	1.4828	1.2743	1.4895	1.2741	1.4962	1.2738	1.5029	1.2735	1.5096	1.2732	1.5164	1.2729
–.17	1.5231	1.2726	1.5300	1.2723	1.5368	1.2720	1.5437	1.2717	1.5506	1.2714	1.5575	1.2710	1.5645	1.2707	1.5715	1.2703	1.5786	1.2699	1.5856	1.2696
–.16	1.5927	1.2692	1.5999	1.2688	1.6070	1.2684	1.6142	1.2680	1.6215	1.2675	1.6288	1.2671	1.6361	1.2667	1.6434	1.2662	1.6508	1.2658	1.6582	1.2653
–.15	1.6657	1.2648	1.6732	1.2644	1.6807	1.2639	1.6883	1.2634	1.6959	1.2629	1.7036	1.2623	1.7113	1.2618	1.7190	1.2613	1.7268	1.2607	1.7346	1.2602
–.14	1.7425	1.2596	1.7504	1.2591	1.7583	1.2585	1.7663	1.2579	1.7744	1.2573	1.7825	1.2567	1.7906	1.2561	1.7988	1.2554	1.8070	1.2548	1.8153	1.2541
–.13	1.8236	1.2535	1.8320	1.2528	1.8404	1.2521	1.8489	1.2515	1.8575	1.2508	1.8660	1.2501	1.8747	1.2493	1.8834	1.2486	1.8921	1.2479	1.9010	1.2471
–.12	1.9098	1.2464	1.9188	1.2456	1.9277	1.2448	1.9368	1.2440	1.9459	1.2432	1.9551	1.2424	1.9643	1.2416	1.9736	1.2408	1.9830	1.2399	1.9924	1.2391
–.11	2.0019	1.2382	2.0115	1.2374	2.0212	1.2365	2.0309	1.2356	2.0407	1.2347	2.0505	1.2338	2.0605	1.2328	2.0705	1.2319	2.0806	1.2309	2.0908	1.2300
–.10	2.1010	1.2290	2.1113	1.2280	2.1218	1.2270	2.1323	1.2260	2.1429	1.2250	2.1536	1.2240	2.1643	1.2229	2.1752	1.2219	2.1862	1.2209	2.1972	1.2197
–.09	2.2084	1.2186	2.2196	1.2175	2.2310	1.2164	2.2425	1.2153	2.2541	1.2141	2.2657	1.2130	2.2775	1.2118	2.2895	1.2106	2.3015	1.2094	2.3136	1.2082
–.08	2.3259	1.2070	2.3383	1.2058	2.3508	1.2045	2.3635	1.2033	2.3763	1.2020	2.3892	1.2007	2.4022	1.1994	2.4155	1.1981	2.4289	1.1967	2.4423	1.1954
–.07	2.4560	1.1940	2.4693	1.1927	2.4838	1.1913	2.4980	1.1898	2.5123	1.1884	2.5268	1.1870	2.5415	1.1855	2.5564	1.1841	2.5715	1.1826	2.5867	1.1811
–.06	2.6022	1.1796	2.6178	1.1780	2.6337	1.1765	2.6499	1.1749	2.6662	1.1733	2.6827	1.1717	2.6995	1.1701	2.7166	1.1684	2.7339	1.1668	2.7516	1.1651
–.05	2.7694	1.1634	2.7876	1.1617	2.8060	1.1599	2.8248	1.1582	2.8439	1.1564	2.8632	1.1546	2.8830	1.1528	2.9031	1.1510	2.9236	1.1491	2.9444	1.1472
–.04	2.9657	1.1453	2.9873	1.1434	3.0095	1.1414	3.0320	1.1395	3.0550	1.1375	3.0786	1.1355	3.1026	1.1334	3.1272	1.1313	3.1523	1.1292	3.1781	1.1271
–.03	3.2044	1.1250	3.2314	1.1228	3.2591	1.1206	3.2876	1.1184	3.3168	1.1161	3.3468	1.1138	3.3777	1.1115	3.4095	1.1091	3.4423	1.1067	3.4761	1.1043
–.02	3.5110	1.1018	3.5473	1.0993	3.5845	1.0968	3.6233	1.0942	3.6636	1.0916	3.7054	1.0889	3.7489	1.0862	3.7945	1.0835	3.8420	1.0807	3.8918	1.0778
–.01	3.9441	1.0749	3.9992	1.0720	4.0574	1.0690	4.1190	1.0659	4.1846	1.0628	4.2546	1.0596	4.3297	1.0563	4.4108	1.0529	4.4988	1.0495	4.5951	1.0460
–.00X	4.7015	1.0423	4.8203	1.0386	4.9548	1.0347	5.1100	1.0307	5.2933	1.0265	5.5175	1.0221	5.8061	1.0174	6.2126	1.0124	6.9068	1.0069	∞	1.0000

APPENDIX C

DETAILS OF SOLUTION OF INTEGRAL (36)

$$F_1 = \int_0^{\epsilon_1} \frac{\frac{\Delta v}{V_o}}{\sqrt{x(c - x)}} \frac{dx}{x - x_o} = \frac{1}{c}\int_0^{\epsilon_1} \frac{\Delta v}{V_o} \frac{1}{\sqrt{\frac{x}{c}}} \frac{1}{\sqrt{1 - \frac{x}{c}}} \frac{d\left(\frac{x}{c}\right)}{\frac{x}{c} - \frac{x_o}{c}} \tag{C1}$$

Introduce $\xi = \frac{x}{c}$ and find

$$F_1 = \frac{1}{c}\int_0^{\epsilon_1/c} \frac{\Delta v}{V_o} \frac{1}{\sqrt{\xi}} \left(1 + \frac{1}{2}\xi + \frac{3}{8}\xi^2 + \cdot \cdot \cdot\right) \frac{d\xi}{\xi - \xi_o} \tag{C2}$$

With the expansion (see equation (37))

$$\frac{\Delta v}{V_o}\left(1 + \frac{1}{2}\xi + \frac{3}{8}\xi^2 + \cdot \cdot \cdot\right) = a_o + \left(a_1 + \frac{1}{2}a_o\right)\xi + \left(a_2 + \frac{1}{2}a_1 + \frac{3}{8}a_o\right)\xi^2$$

$$= a_o + a_1^*\xi + a_2^*\xi^2 + \cdot \cdot \cdot \tag{C3}$$

Hence,

$$F_1 = \frac{1}{c}\int_0^{\epsilon_1/c} \frac{a_o + a_1^*\xi + a_2^*\xi^2}{\sqrt{\xi}\left(\xi - \xi_o\right)} d\xi$$

$$= \frac{1}{c}\left[a_o \int_0^{\epsilon_1/c} \frac{d\xi}{\sqrt{\xi}\left(\xi - \xi_o\right)} + a_1^* \int_0^{\epsilon_1/c} \frac{\xi\, d\xi}{\sqrt{\xi}\left(\xi - \xi_o\right)} + a_2^* \int_0^{\epsilon_1/c} \frac{\xi^2\, d\xi}{\sqrt{\xi}\left(\xi - \xi_o\right)}\right] \tag{C4}$$

As the occurring integrals are all of the same type, define

$$L_n = \int_0^{\epsilon_1/c} \frac{\xi^n \, d\xi}{\sqrt{\xi}\left(\xi - \xi_o\right)} \tag{C5}$$

These integrals L_n are easily solved by recurrence.

$$L_n = \xi_o L_{n-1} + \frac{\left(\frac{\epsilon_1}{c}\right)^{n-\frac{1}{2}}}{n - \frac{1}{2}} \tag{C6}$$

with

$$L_o = \frac{1}{\sqrt{\xi_o}} \log_e \frac{1 - \sqrt{\frac{\epsilon_1}{x_o}}}{1 + \sqrt{\frac{\epsilon_1}{x_o}}} \quad \text{for} \quad x_o > \epsilon_1 \tag{C7}$$

and

$$L_o = \frac{1}{\sqrt{\xi_o}} \log_e \frac{1 - \sqrt{\frac{x_o}{\epsilon_1}}}{1 + \sqrt{\frac{x_o}{\epsilon_1}}} \quad \text{for} \quad x_o < \epsilon_1 \tag{C8}$$

The function

$$M_o = \log_e \frac{1 - \sqrt{\frac{\epsilon_1}{x_o}}}{1 + \sqrt{\frac{\epsilon_1}{x_o}}} \quad \text{and} \quad \log_e \frac{1 - \sqrt{\frac{x_o}{\epsilon_1}}}{1 + \sqrt{\frac{x_o}{\epsilon_1}}}$$

is given in figure 2 in order to provide a more rapid computation in the event that $\frac{x_o}{\epsilon_1}$ or $\frac{\epsilon_1}{x_o}$ is not very small.

If $\frac{\epsilon_1}{x_o} \ll 1$,

$$M_o = -2\left(\sqrt{\frac{\epsilon_1}{x_o}} + \frac{1}{3}\sqrt{\frac{\epsilon_1}{x_o}}^{\,3} + \frac{1}{5}\sqrt{\frac{\epsilon_1}{x_o}}^{\,5} + \cdot \cdot \cdot\right) \tag{C9}$$

The integrals L_o, L_1, and L_2 are needed; these are given by

$$\left.\begin{aligned} L_o &= \frac{1}{\sqrt{\xi_o}} M_o \\ L_1 &= \sqrt{\xi_o} M_o + 2\sqrt{\frac{\epsilon_1}{c}} \\ L_2 &= \xi_o L_1 + \frac{2}{3}\sqrt{\frac{\epsilon_1}{c}}^{\,3} \\ &= \xi_o^{\,3/2} M_o + 2\xi_o\sqrt{\frac{\epsilon_1}{c}} + \frac{2}{3}\sqrt{\frac{\epsilon_1}{c}}^{\,3} \end{aligned}\right\} \tag{C10}$$

If $\frac{\epsilon_1}{x_o} \ll 1$,

$$\left.\begin{aligned} L_o &= -\frac{2}{\sqrt{\xi_o}}\left(\sqrt{\frac{\epsilon_1}{x_o}} + \frac{1}{3}\sqrt{\frac{\epsilon_1}{x_o}}^{\,3} + \frac{1}{5}\sqrt{\frac{\epsilon_1}{x_o}}^{\,5} + \cdot \cdot \cdot\right) \\ L_1 &= -2\sqrt{\xi_o}\left(\frac{1}{3}\sqrt{\frac{\epsilon_1}{x_o}}^{\,3} + \frac{1}{5}\sqrt{\frac{\epsilon_1}{x_o}}^{\,5} + \cdot \cdot \cdot\right) \\ L_2 &= -2\sqrt{\xi_o}^{\,3}\left(\frac{1}{5}\sqrt{\frac{\epsilon_1}{x_o}}^{\,5} + \cdot \cdot \cdot\right) \end{aligned}\right\} \tag{C11}$$

With these expressions the integral F_1 is as follows:

$$F_1 = \frac{1}{c}\left(a_o L_o + a_1{}^* L_1 + a_2{}^* L_2\right)$$

$$= \frac{1}{c}\left[\frac{a_o}{\sqrt{\xi_o}} M_o + a_1{}^*\left(\sqrt{\xi_o}\, M_o + 2\sqrt{\frac{\epsilon_1}{c}}\right) + a_2{}^*\left(\sqrt{\xi_o}^{\,3} M_o + 2\xi_o\sqrt{\frac{\epsilon_1}{c}} + \frac{2}{3}\sqrt{\frac{\epsilon_1}{c}}^{\,3}\right)\right]$$

$$= \frac{1}{c}\left[M_o\left(\frac{a_o}{\sqrt{\xi_o}} + a_1{}^*\sqrt{\xi_o} + a_2{}^*\sqrt{\xi_o}^{\,3}\right) + 2\sqrt{\frac{\epsilon_1}{c}}\left(a_1{}^* + \xi_o a_2{}^*\right) + \frac{2}{3} a_2{}^*\sqrt{\frac{\epsilon_1}{c}}^{\,3}\right] \tag{C12}$$

The coefficients a_o, a_1, and a_2 of the expansion of $\frac{\Delta v}{V_o}$ are given by

$$\left.\begin{aligned}
a_o &= \left(\frac{\Delta v}{V_o}\right)_{x=0} \\
a_1 &= \frac{c}{2\epsilon_1}\left[-3\left(\frac{\Delta v}{V_o}\right)_{x=0} + 4\left(\frac{\Delta v}{V_o}\right)_{x=\epsilon_1} - \left(\frac{\Delta v}{V_o}\right)_{x=2\epsilon_1}\right] \\
a_2 &= \frac{c^2}{2\epsilon_1{}^2}\left[\left(\frac{\Delta v}{V_o}\right)_{x=0} - 2\left(\frac{\Delta v}{V_o}\right)_{x=\epsilon_1} + \left(\frac{\Delta v}{V_o}\right)_{x=2\epsilon_1}\right]
\end{aligned}\right\} \tag{C13}$$

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TABLE I.- VALUES OF j_{no} AND j_{no}^* FOR $-49.5 < \frac{x_n - x_o}{\Delta x} < 49.5$

$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*	$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*
-49.5	-0.0204	-0.0102	0.5	1.0986	0.4507
-48.5	-.0208	-.0104	1.5	.5108	.2338
-47.5	-.0213	-.0107	2.5	.3365	.1588
-46.5	-.0217	-.0109	3.5	.2513	.1204
-45.5	-.0222	-.0112	4.5	.2007	.0970
-44.5	-.0227	-.0114	5.5	.1671	.0812
-43.5	-.0233	-.0117	6.5	.1431	.0698
-42.5	-.0238	-.0120	7.5	.1252	.0613
-41.5	-.0244	-.0122	8.5	.1112	.0546
-40.5	-.0250	-.0125	9.5	.1001	.0492
-39.5	-.0256	-.0129	10.5	.0910	.0448
-38.5	-.0263	-.0132	11.5	.0834	.0411
-37.5	-.0270	-.0136	12.5	.0770	.0380
-36.5	-.0278	-.0139	13.5	.0715	.0353
-35.5	-.0286	-.0143	14.5	.0667	.0330
-34.5	-.0294	-.0148	15.5	.0625	.0309
-33.5	-.0303	-.0153	16.5	.0588	.0291
-32.5	-.0313	-.0157	17.5	.0556	.0275
-31.5	-.0323	-.0162	18.5	.0526	.0261
-30.5	-.0333	-.0167	19.5	.0500	.0248
-29.5	-.0345	-.0173	20.5	.0476	.0236
-28.5	-.0357	-.0180	21.5	.0455	.0226
-27.5	-.0370	-.0186	22.5	.0435	.0216
-26.5	-.0385	-.0193	23.5	.0417	.0207
-25.5	-.0400	-.0201	24.5	.0400	.0199
-24.5	-.0417	-.0210	25.5	.0385	.0191
-23.5	-.0435	-.0219	26.5	.0370	.0184
-22.5	-.0455	-.0229	27.5	.0357	.0178
-21.5	-.0476	-.0240	28.5	.0345	.0172
-20.5	-.0500	-.0252	29.5	.0333	.0166
-19.5	-.0526	-.0266	30.5	.0323	.0160
-18.5	-.0556	-.0280	31.5	.0313	.0155
-17.5	-.0588	-.0297	32.5	.0303	.0151
-16.5	-.0625	-.0316	33.5	.0294	.0146
-15.5	-.0667	-.0337	34.5	.0286	.0142
-14.5	-.0715	-.0362	35.5	.0278	.0138
-13.5	-.0770	-.0390	36.5	.0270	.0134
-12.5	-.0834	-.0423	37.5	.0263	.0131
-11.5	-.0910	-.0462	38.5	.0256	.0127
-10.5	-.1001	-.0509	39.5	.0250	.0125
-9.5	-.1112	-.0567	40.5	.0244	.0122
-8.5	-.1252	-.0639	41.5	.0238	.0118
-7.5	-.1431	-.0733	42.5	.0233	.0116
-6.5	-.1671	-.0859	43.5	.0227	.0113
-5.5	-.2007	-.1037	44.5	.0222	.0111
-4.5	-.2513	-.1309	45.5	.0217	.0108
-3.5	-.3365	-.1777	46.5	.0213	.0106
-2.5	-.5108	-.2771	47.5	.0208	.0103
-1.5	-1.0986	-.6479	48.5	.0204	.0101
-.5	0	1.0	49.5	.0200	.0100

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TABLE II.- COMPUTATION BY UNEQUAL INTERVALS, TRANSITION FROM ONE INTERVAL SIZE TO ANOTHER

(a) $\overline{\Delta x} = 0.002$.

$\frac{x}{c}$	σ_n	$\sigma_{n+1} - \sigma_n$	$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*
0	σ_0	$\sigma_1 - \sigma_0$	-4.5	-0.2513	-0.1309
.002	σ_1	$\sigma_2 - \sigma_1$	-3.5	-.3365	-.1777
.004	σ_2	$\sigma_3 - \sigma_2$	-2.5	-.5108	-.2771
.006	σ_3	$\sigma_4 - \sigma_3$	-1.5	-1.0986	-.6479
.008	σ_4	$\sigma_5 - \sigma_4$	-.5	0	1.0
.010	σ_5	$\sigma_6 - \sigma_5$	.5	1.0986	.4507
.012	σ_6	$\sigma_7 - \sigma_6$	1.5	.5108	.2338
.014	.	.	2.5	.3365	.1588
.016	.	.	3.5	.2513	.1204
.018	.	.	4.5	.2007	.0970
.020			5.5	.1671	.0812
.022			6.5	.1431	.0698
.024	.	.	7.5	.1252	.0613
.026	.	.	8.5	.1112	.0546
.028	σ_{14}	$\sigma_{15} - \sigma_{14}$	9.5	.1001	.0492
.030	σ_{15}		10.5		

(b) $\overline{\Delta x} = 0.006$.

$\frac{x}{c}$	σ_n	$\sigma_{n+1} - \sigma_n$	$\frac{x_n - x_o}{\Delta x}$	j_{no}	j_{no}^*
0	σ_0	$\sigma_3 - \sigma_0$	-1.5	-1.0986	-0.6479
.006	σ_3	$\sigma_6 - \sigma_3$	-.5	0	1.0
.012	σ_6	$\sigma_9 - \sigma_6$	.5	1.0986	.4507
.018	σ_9	$\sigma_{12} - \sigma_9$	1.5	.5108	.2338
.024	σ_{12}	$\sigma_{15} - \sigma_{12}$	2.5	.3365	.1588
.030	σ_{15}	$\sigma_{16} - \sigma_{15}$	3.5	.2513	.1204
.036	σ_{16}	$\sigma_{17} - \sigma_{16}$	4.5	.2007	.0970
.042	σ_{17}	$\sigma_{18} - \sigma_{17}$	5.5	.1671	.0812
.048	σ_{18}	$\sigma_{19} - \sigma_{18}$	6.5	.1431	.0698
.054	.	.	7.5	.1252	.0613
.060	.	.	8.5	.1112	.0546
.066	.	.	9.5	.1001	.0492
.072			10.5	.0910	.0448
.078	.	.	11.5	.0834	.0411
.084	.	.	12.5	.0770	.0380
.090	σ_{25}	$\sigma_{26} - \sigma_{25}$	13.5	.0715	.0353
.096	σ_{26}	$\sigma_{27} - \sigma_{26}$	14.5	.0667	.0330

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TABLE III.- COMPUTATION FOR $x_o = 0.065$ BY UNEQUAL INTERVALS

[Example, fig. 18]

$\frac{x}{c}$	σ_n	$\sigma_{n+1} - \sigma_n$	$\frac{x_n - x_o}{\overline{\Delta x}_n}$	$\frac{x_n - x_o}{\overline{\overline{\Delta x}}_n}$	$\frac{x_n - x_o}{\overline{\overline{\overline{\Delta x}}}_n}$	j_{no}	j_{no}^*
0.060	0	0.005	-1.5			-1.099	-0.648
.0633	.005	.010	-.5			0	1.0
.0667	.015	.021	.5			1.099	.451
.070	.036	.0565		1.0		.693	.307
.075	.0925	.1075		2.0		.406	.189
.080	.2000	-.1000		3.0		.288	.137
.085	.1000	-.094		4.0		.223	.107
.090	.006	-.0087			2.5	.336	.159
.100	-.0027	.0011			3.5	.251	.120
.110	-.0016	.0016			4.5	.201	.097
.120	0	0			5.5	.167	.081
			$\overline{\Delta x} = 0.0033$	$\overline{\overline{\Delta x}} = 0.005$	$\overline{\overline{\overline{\Delta x}}} = 0.010$		

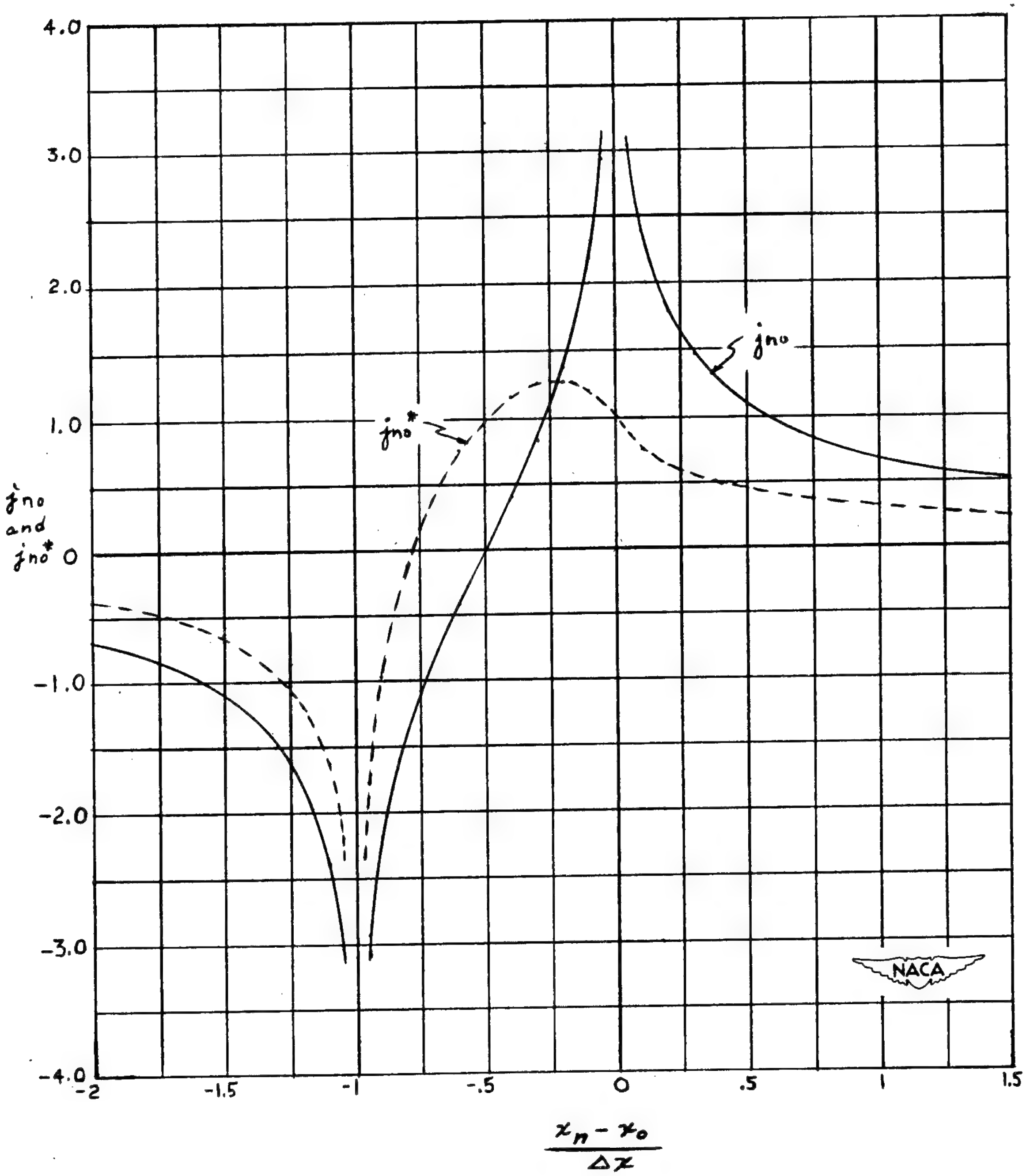


Figure 1.- Characteristic qualities of j_{no} and j_{no}^* as functions of $\frac{x_n - x_o}{\Delta x}$.

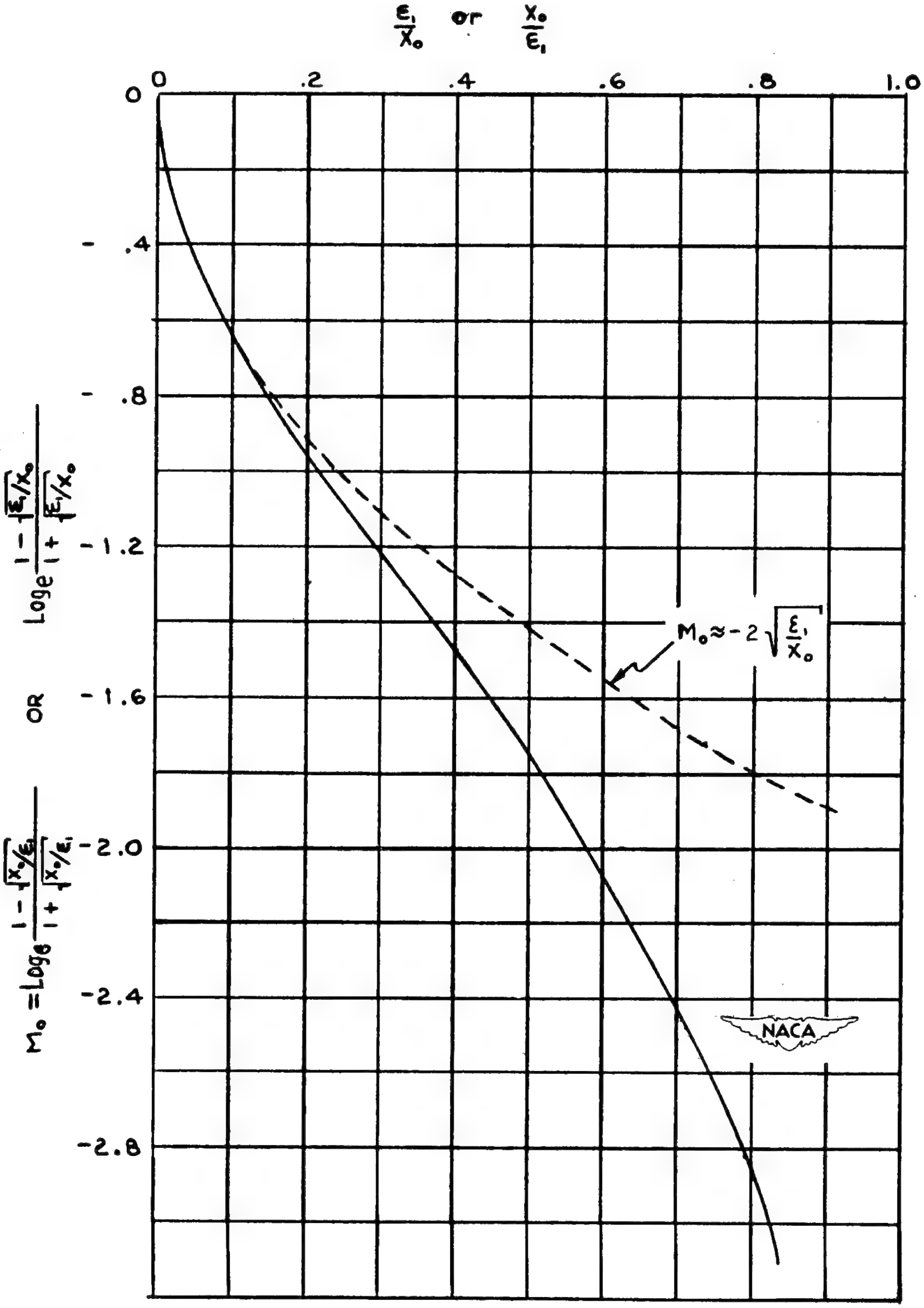
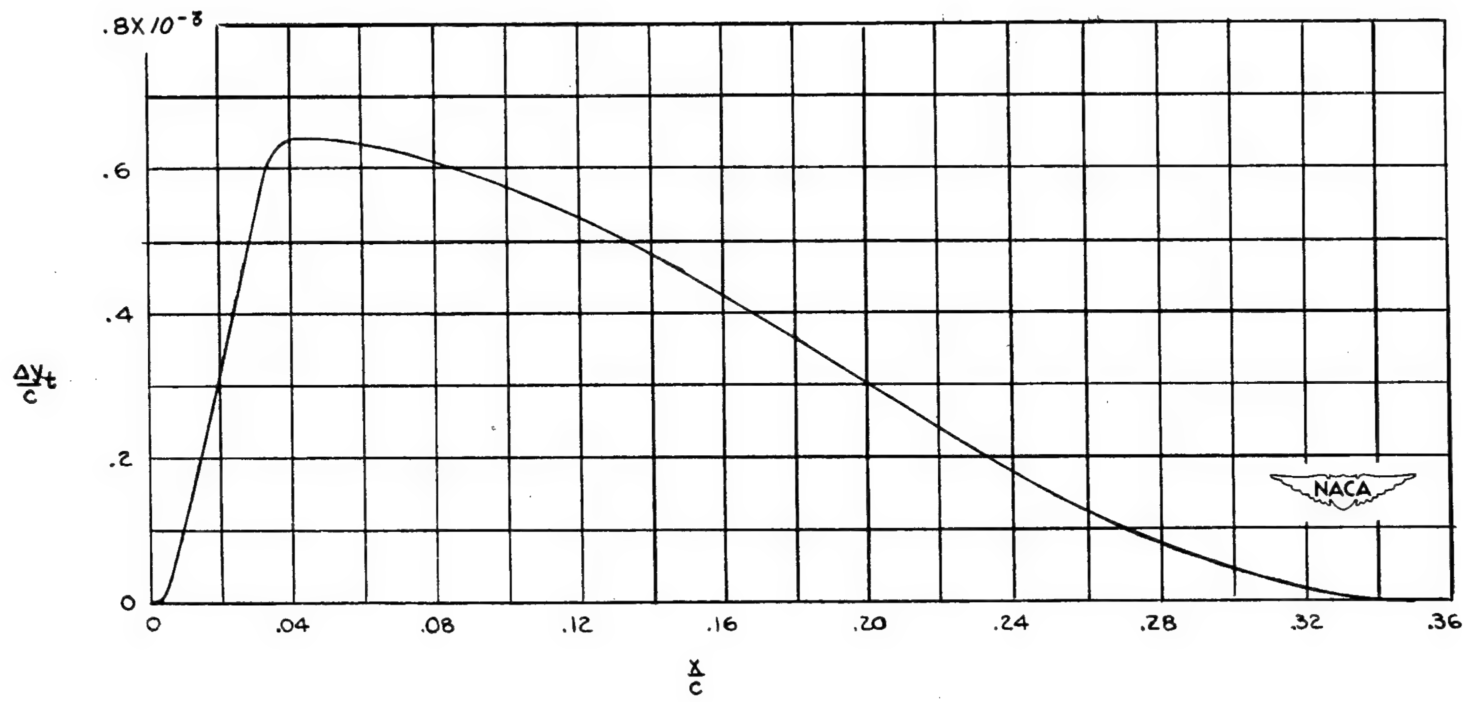
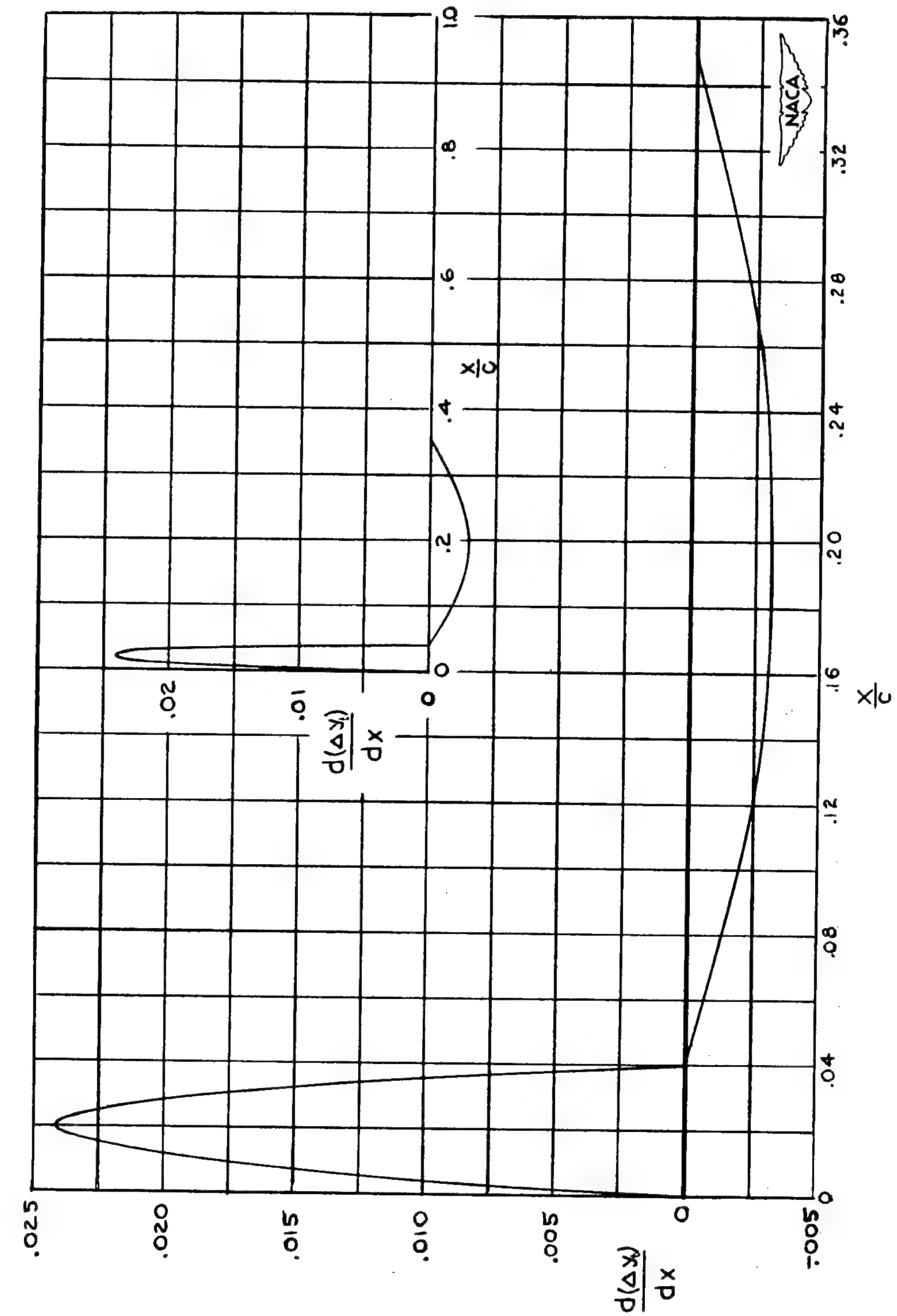


Figure 2.- Function M_0 for computation when x_0/ϵ_1 or ϵ_1/x_0 is not small.

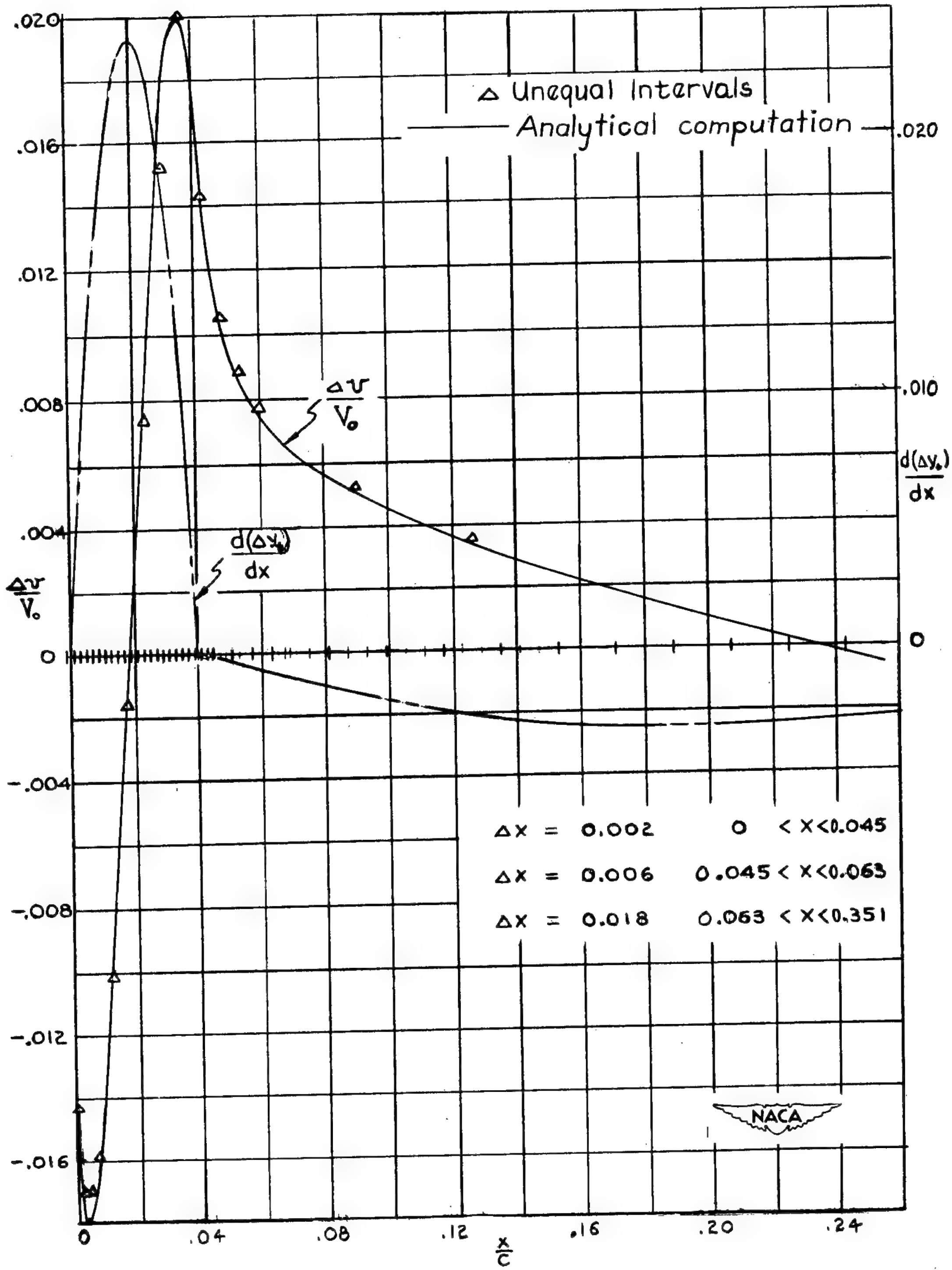


(a) Function Δy_t.

Figure 3.- Analytical example for testing accuracy of method.

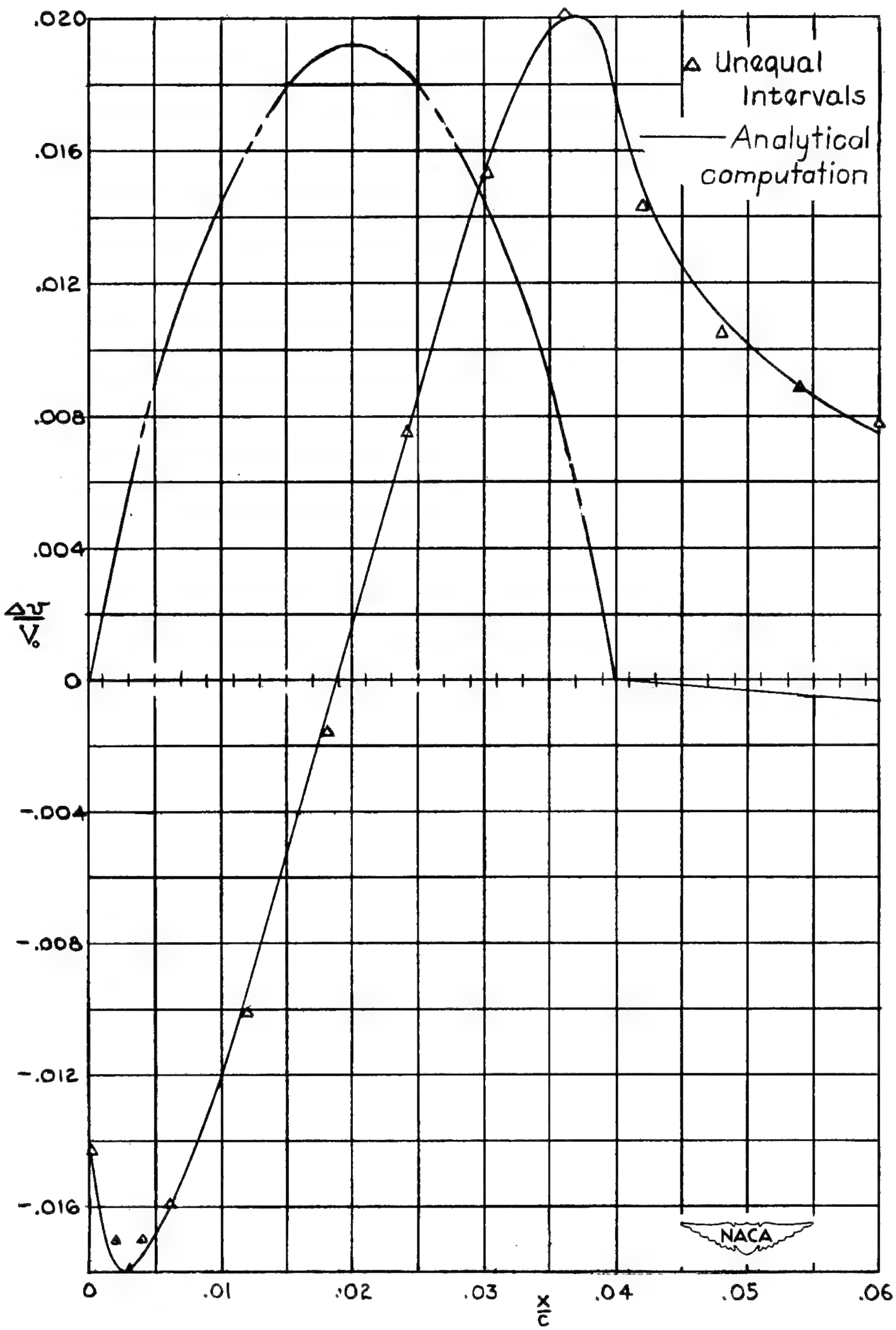


(b) Function $\frac{d(\Delta y_t)}{dx}$.



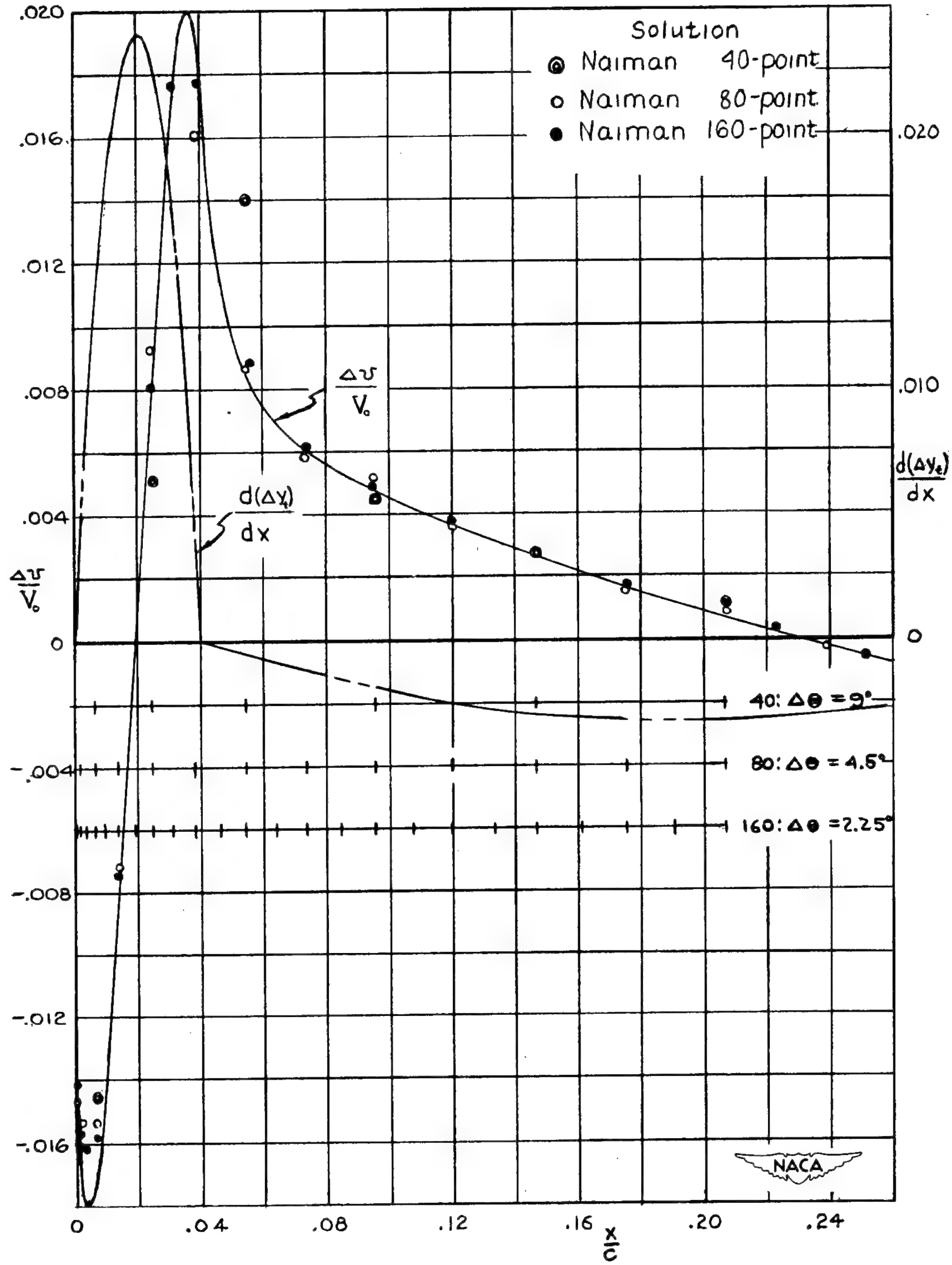
(a) Plot for $0 < x < 0.25$.

Figure 4.- Analytical computation of $\frac{\Delta v}{V_o}$ for figure 3(b) and comparison with results by computation with unequal intervals.



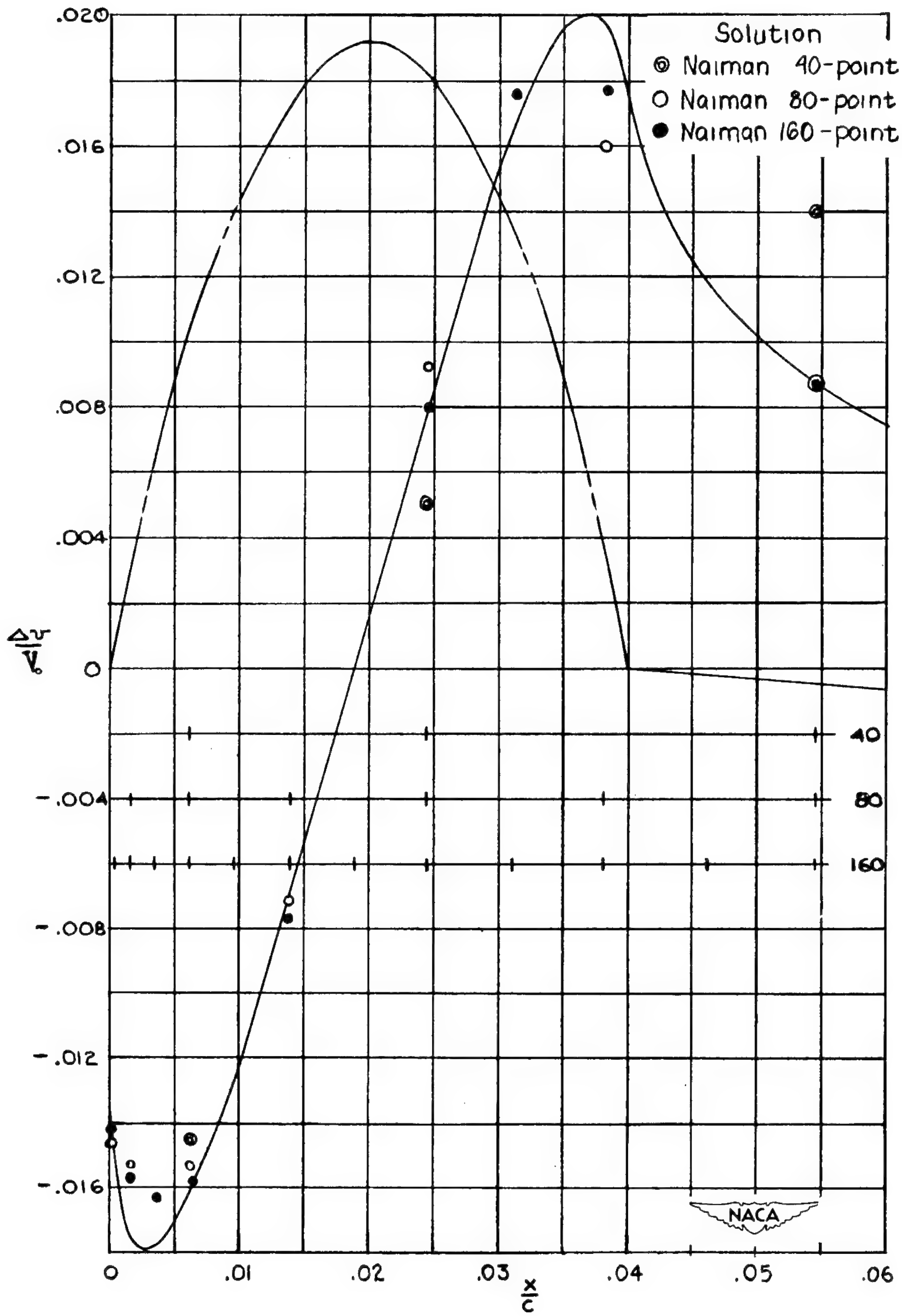
(b) Part of figure 4(a) plotted to larger scale.

Figure 4.- Concluded.



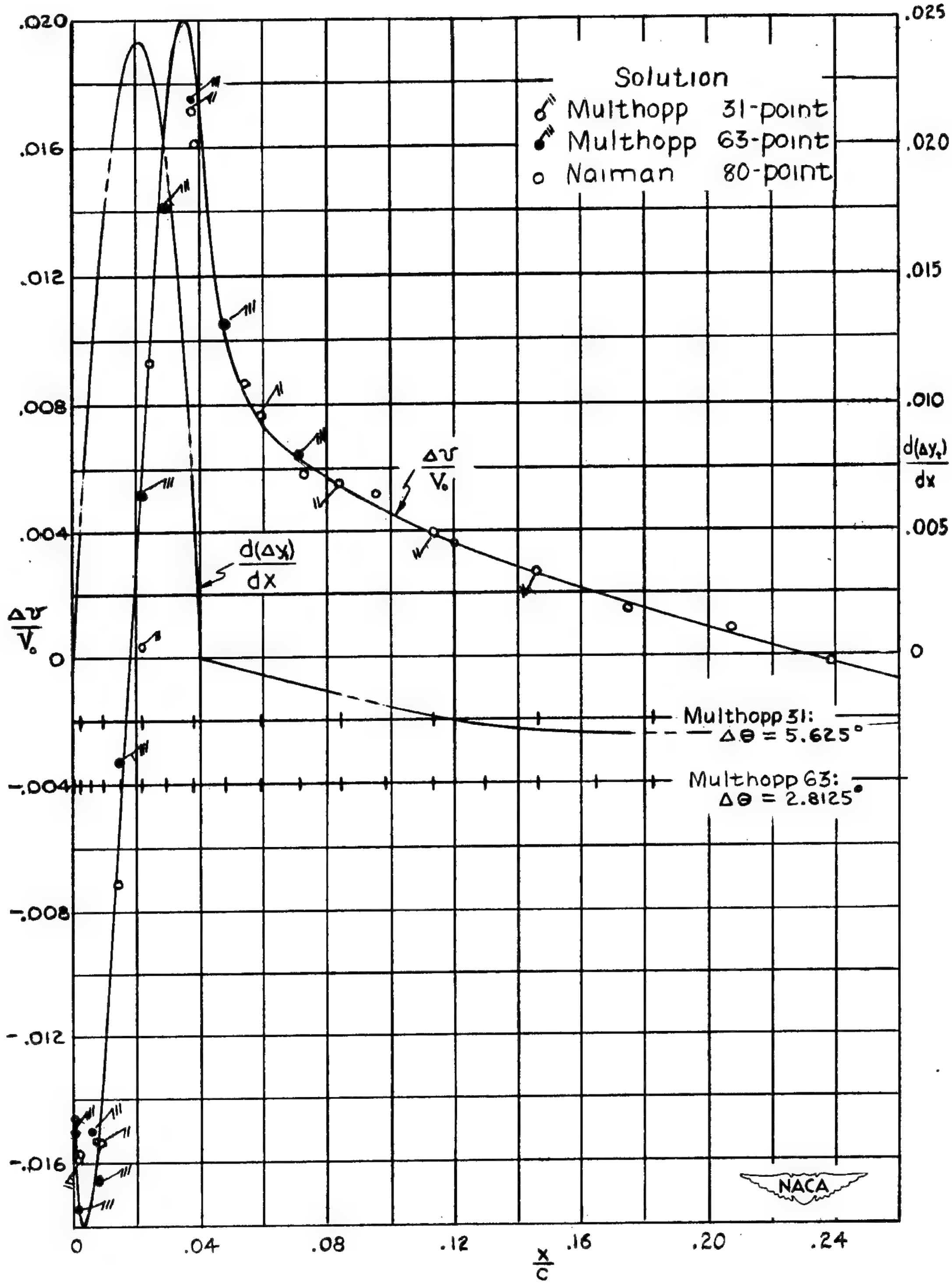
(a) Comparison with analytical results.

Figure 5.- Results obtained by Naiman's method. 40-, 80-, and 160-point solutions.



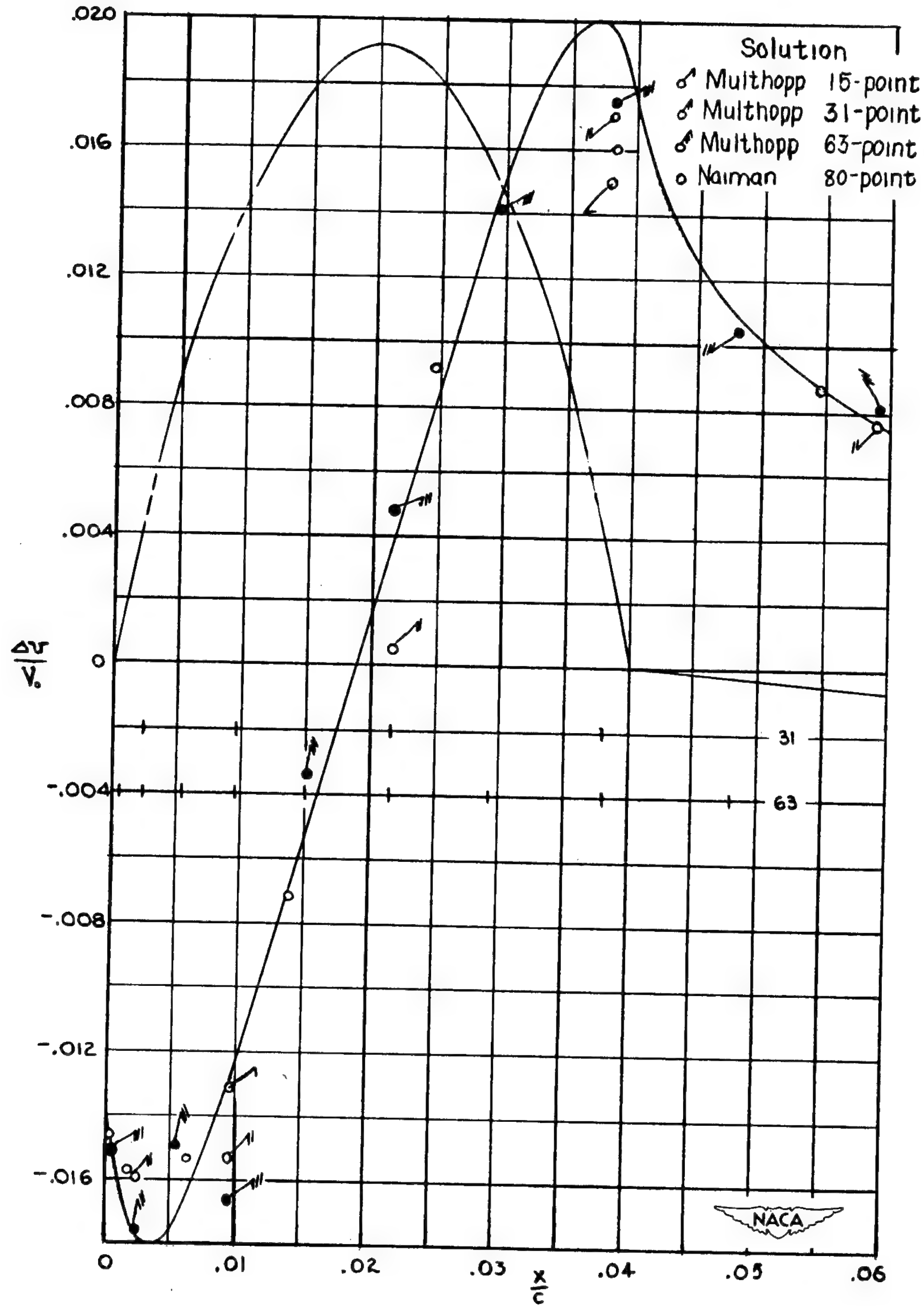
(b) Figure 5(a) plotted to a larger scale.

Figure 5.- Concluded.



(a) Comparison with analytical results and results of Naiman's method.

Figure 6.- Results obtained by Multhopp's method. 31- and 63-point solutions.



(b) Figure 6(a) plotted to larger scale.

Figure 6.- Concluded.

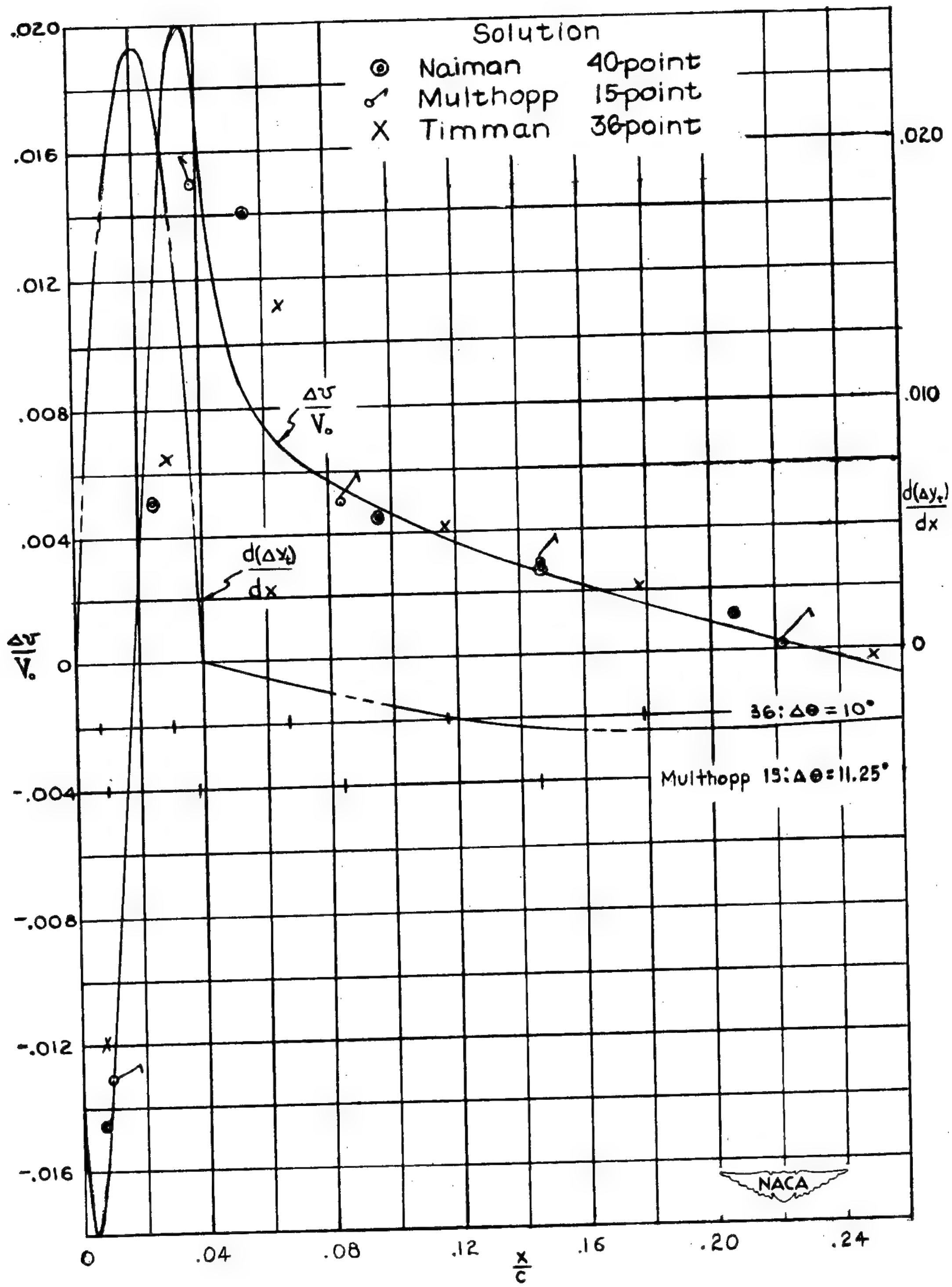


Figure 7.- Comparison of methods of Naiman, Multhopp, and Timman with analytical solution as basis.

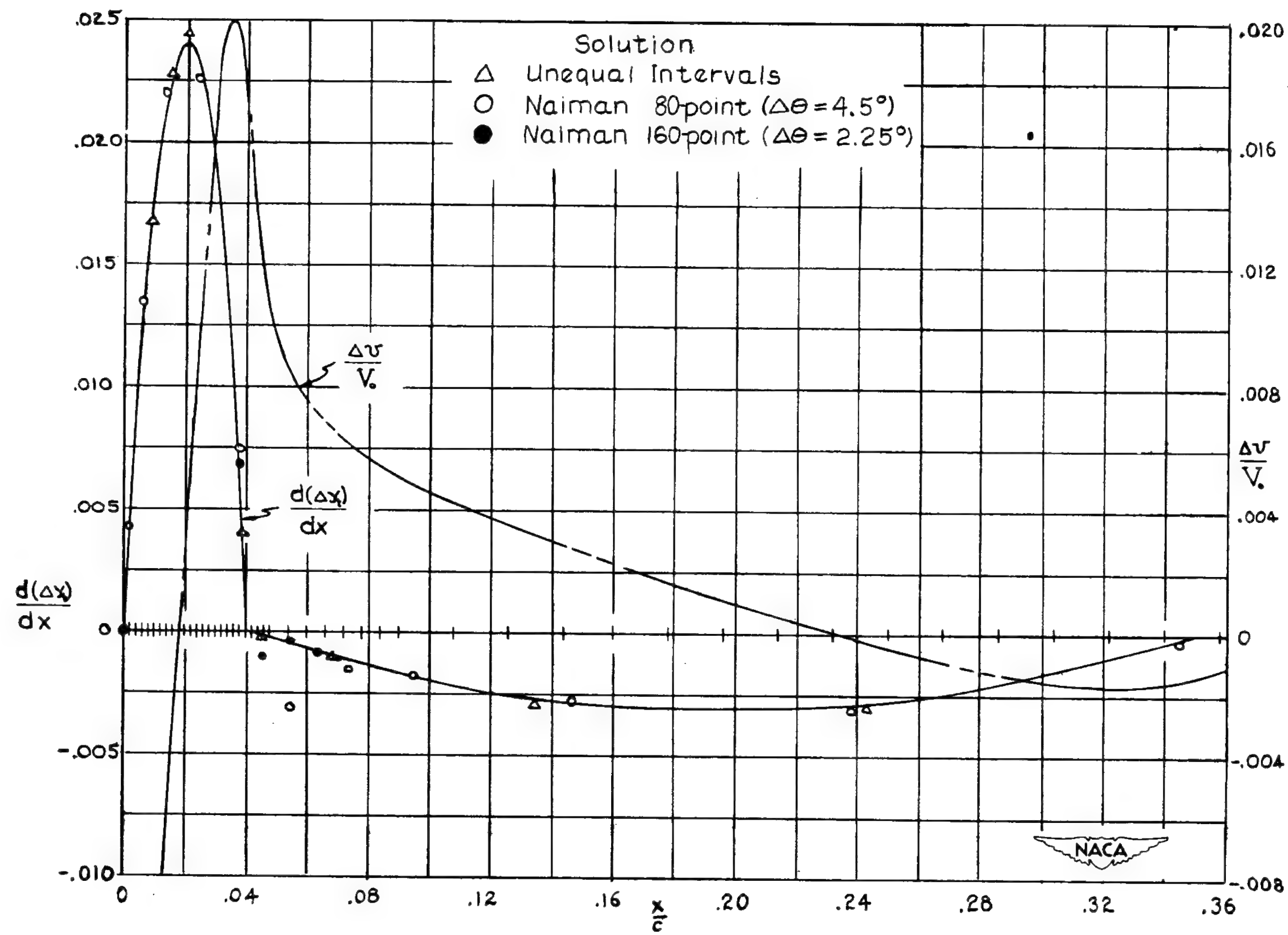
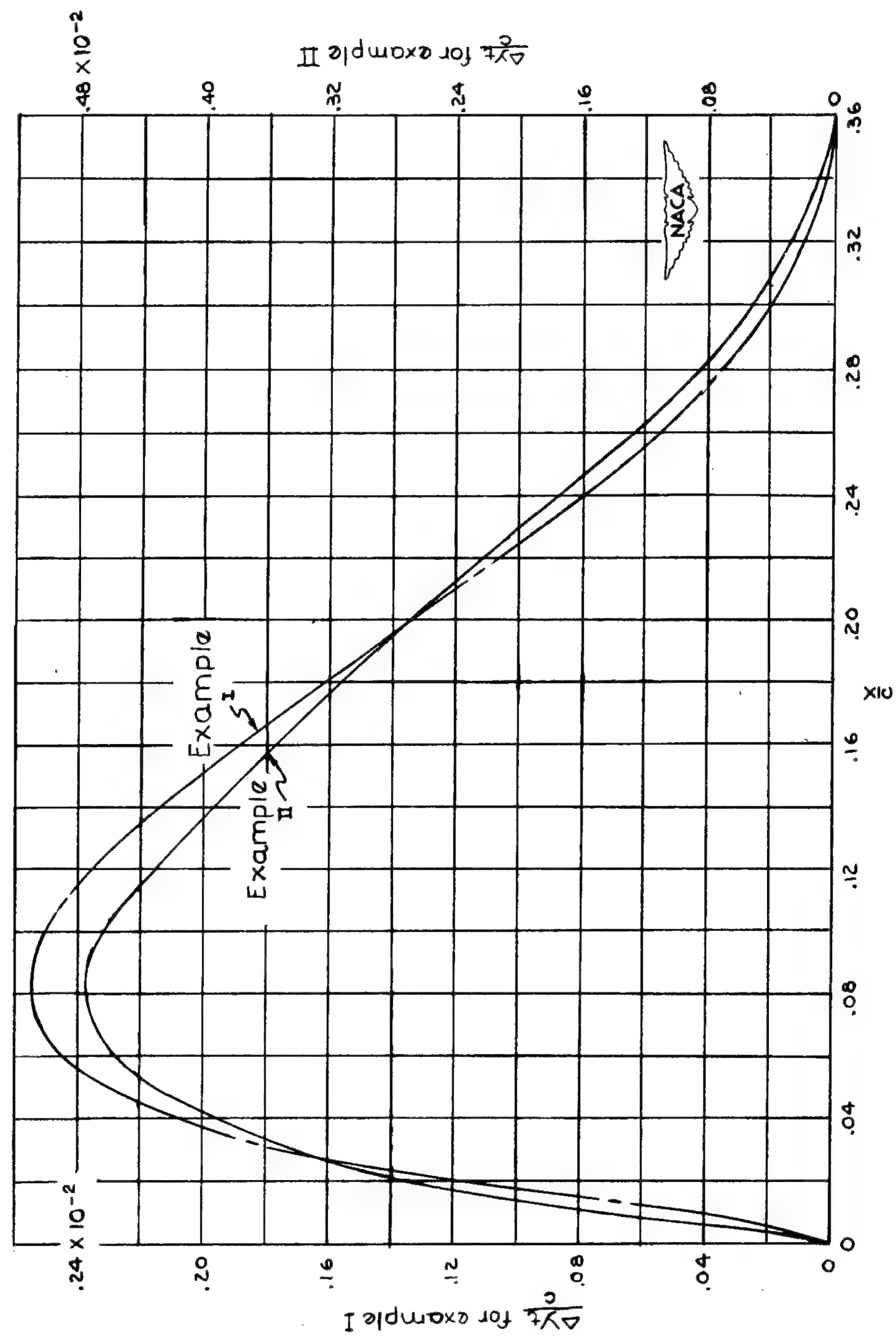
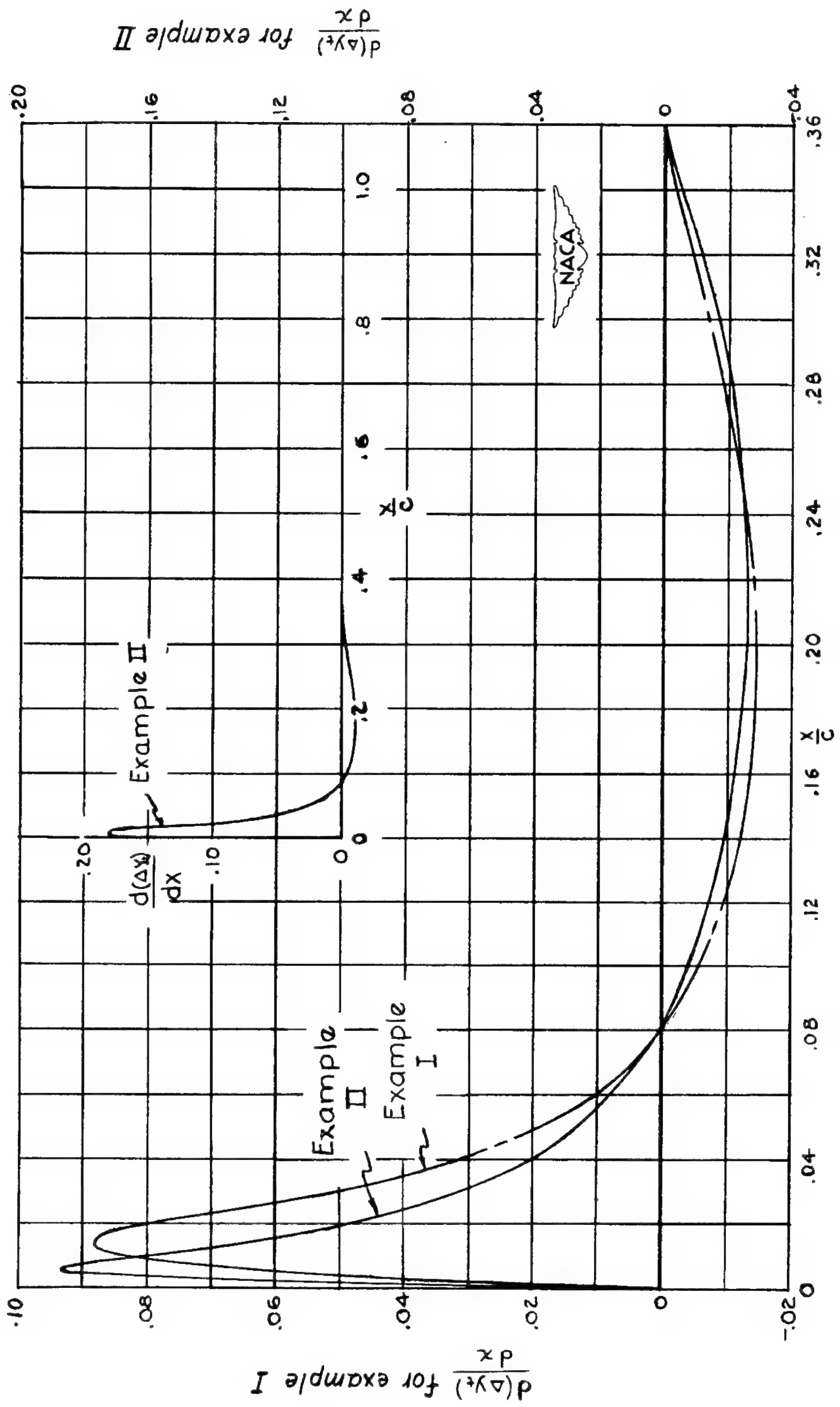


Figure 8.- Solution of inverse problem. Comparison of Naiman's 80- and 160-point solutions with that obtained by the method of unequal intervals.



(a) Functions $\Delta y_t(x)$.

Figure 9.- Examples I and II.



(b) $\frac{d(\Delta y_t)}{dx}$ as function of x/c.

Figure 9.- Concluded.

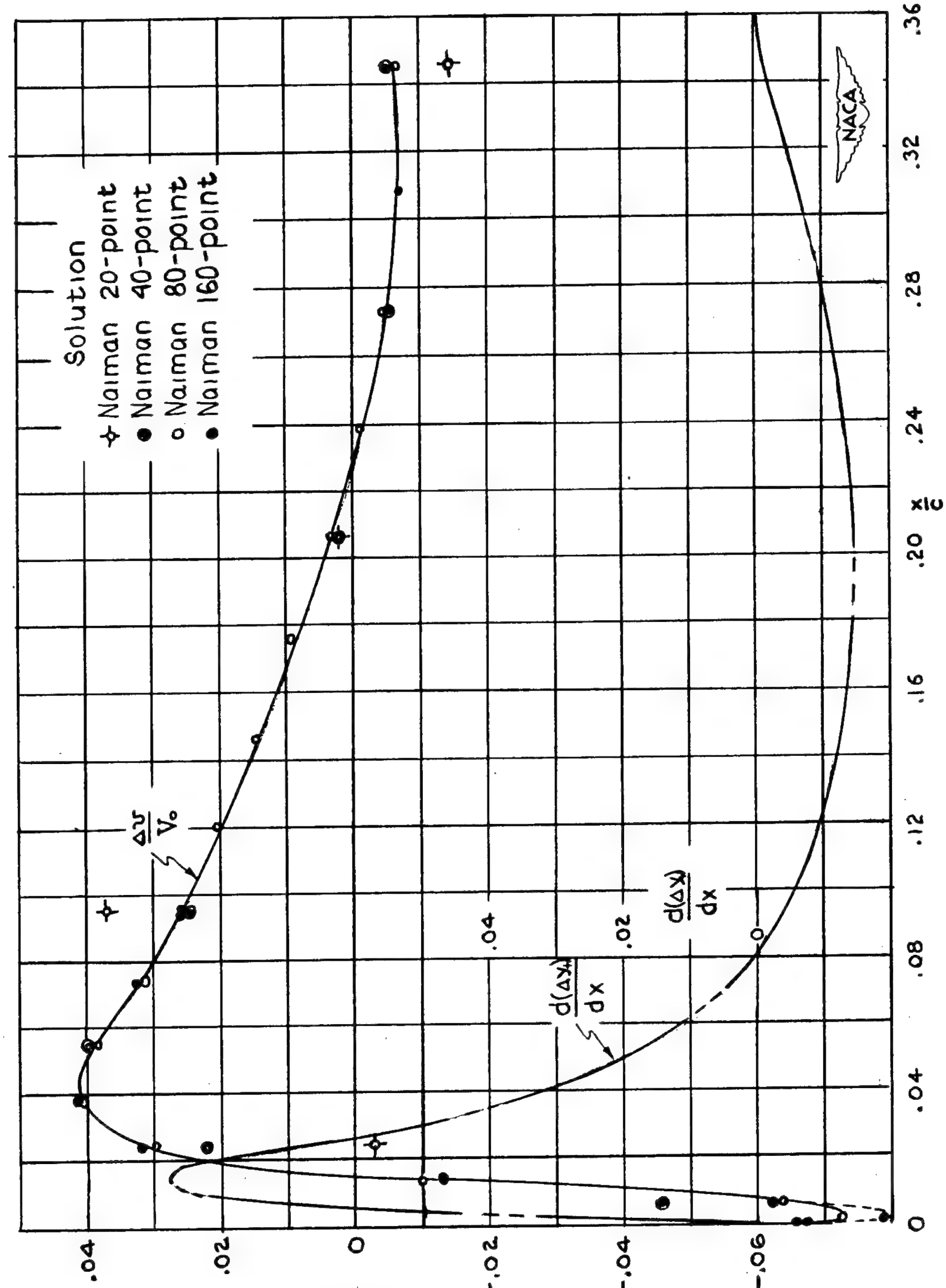


Figure 10.- Direct problem for example I by Naiman's method.

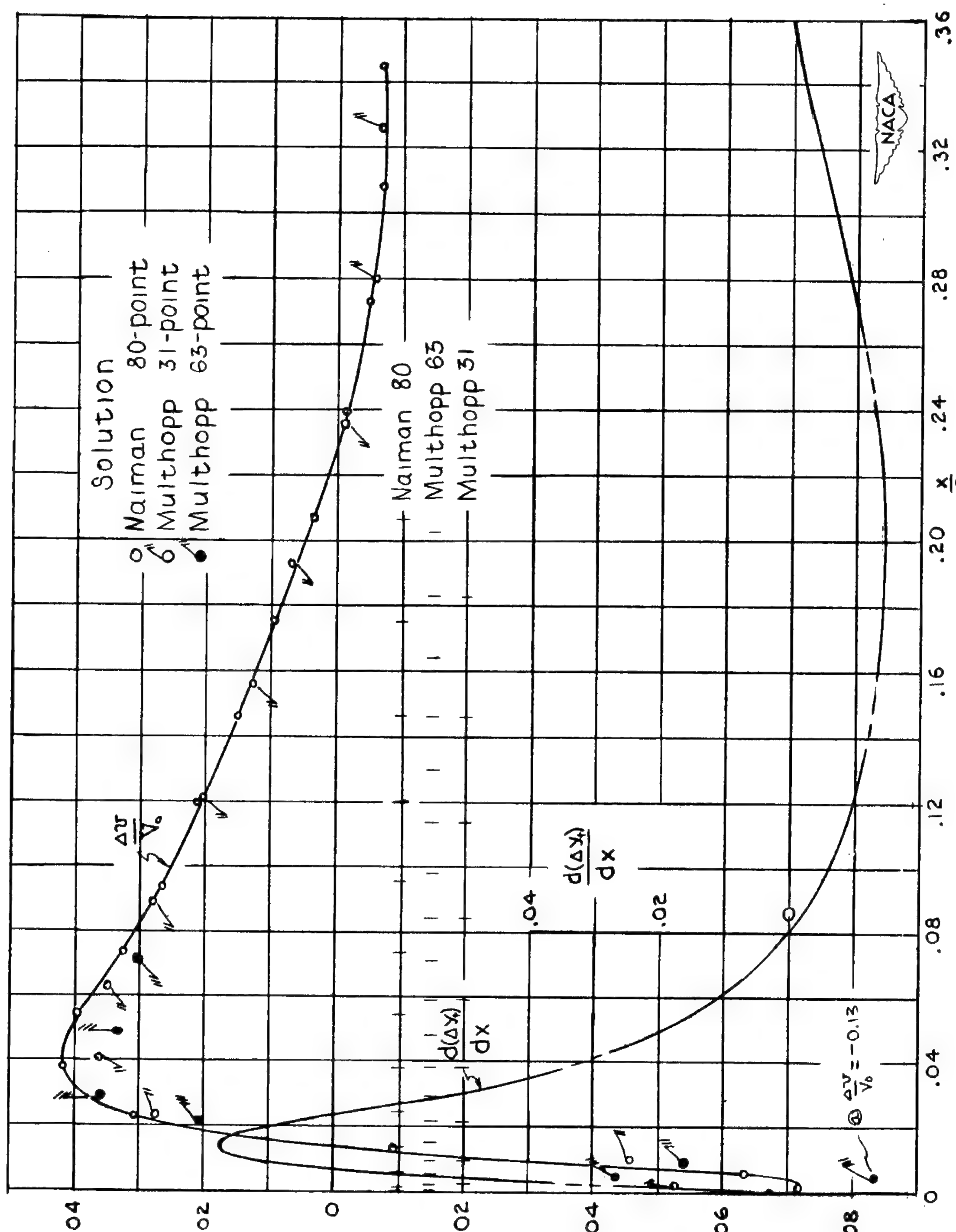


Figure 11.- Direct problem for example I by Multhopp's method.

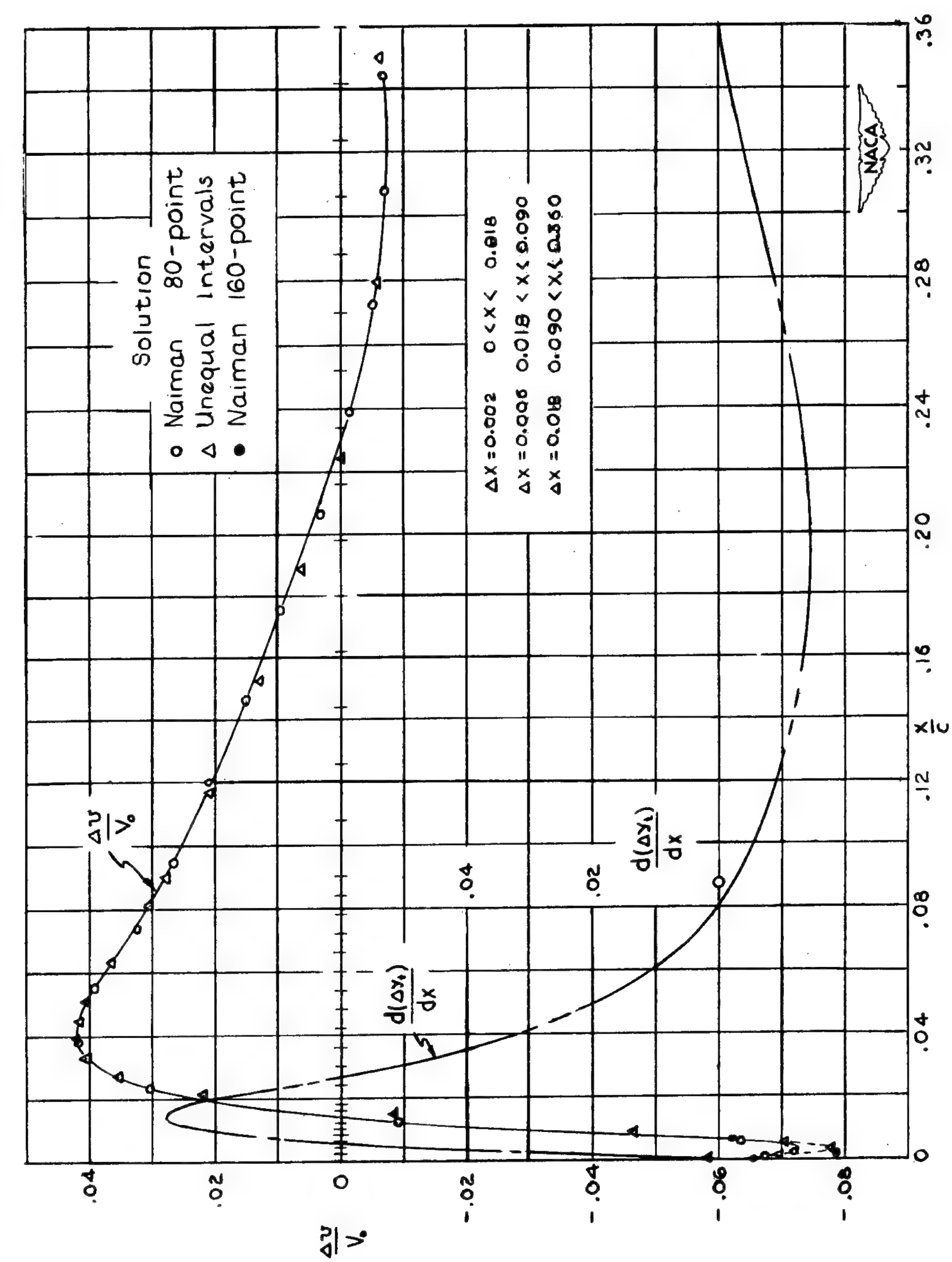


Figure 12.- Results obtained for example I by method of unequal intervals compared with results obtained by Naiman's 80- and 160-point solutions.

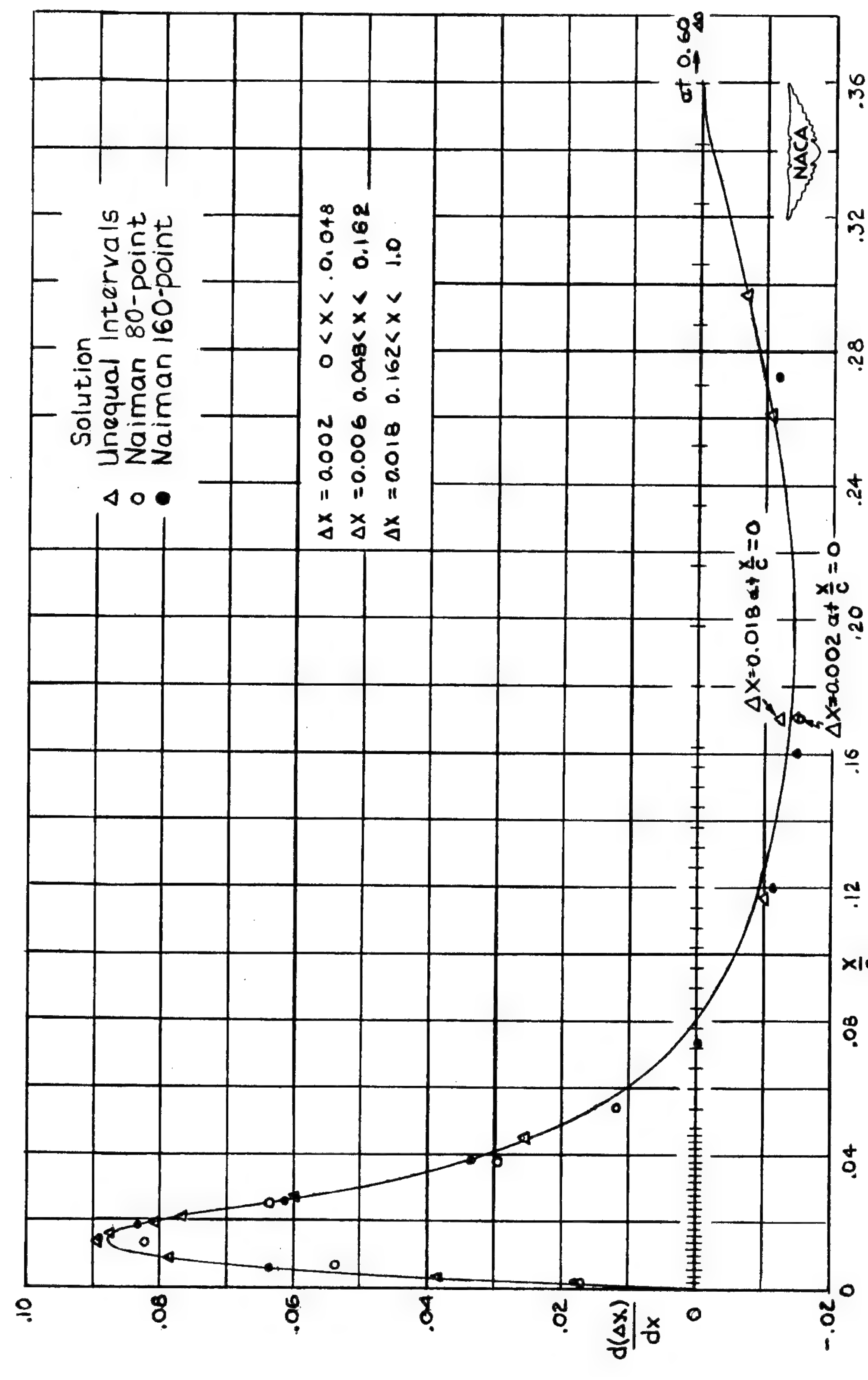


Figure 13.- Solution of inverse problem. Results obtained for example I by method of unequal intervals compared with results obtained by Naiman's 80- and 160-point solutions.

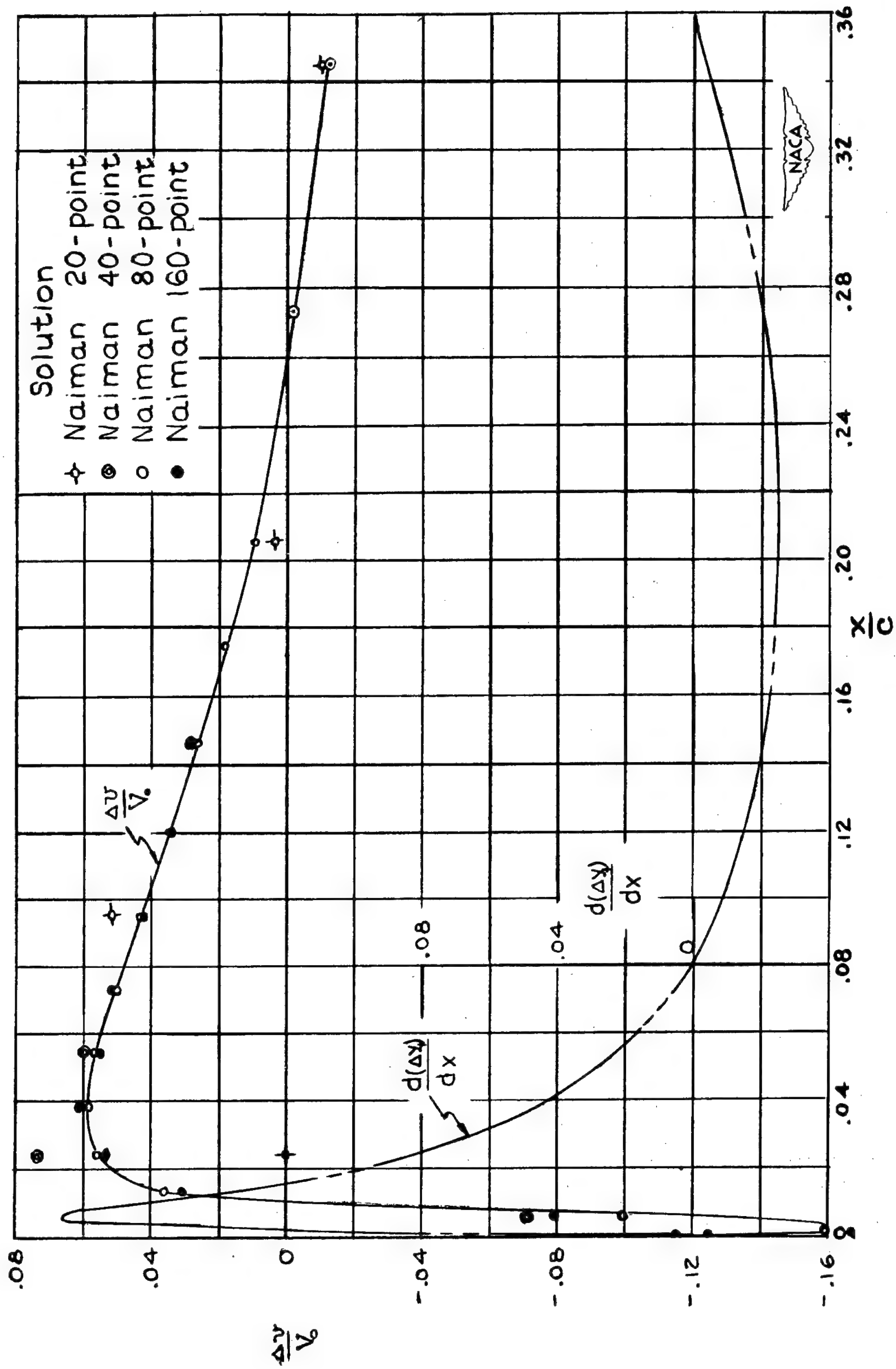


Figure 14.- Direct problem for example II by Naiman's method.

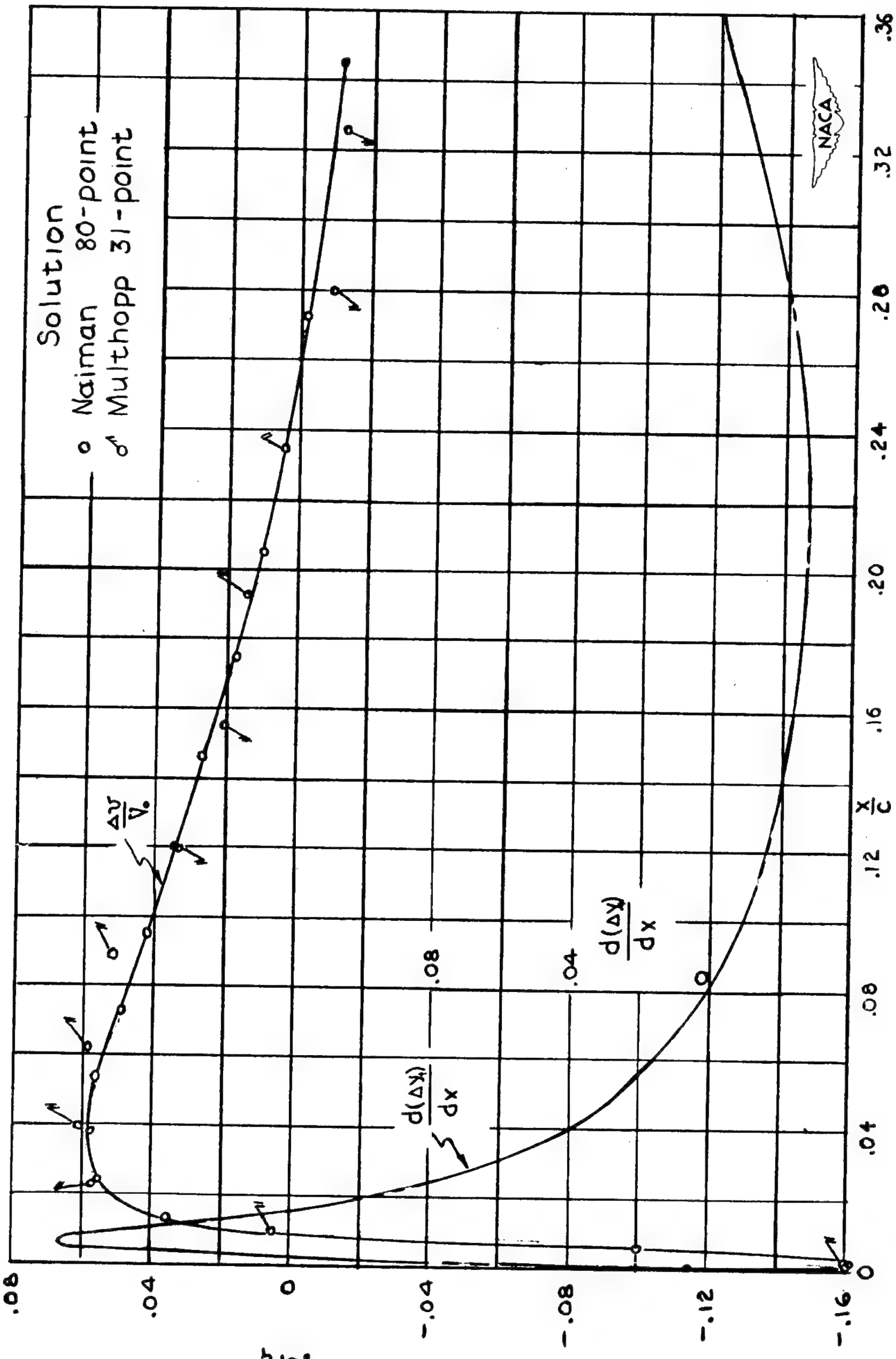


Figure 15.- Direct problem for example II by Multhopp's method.

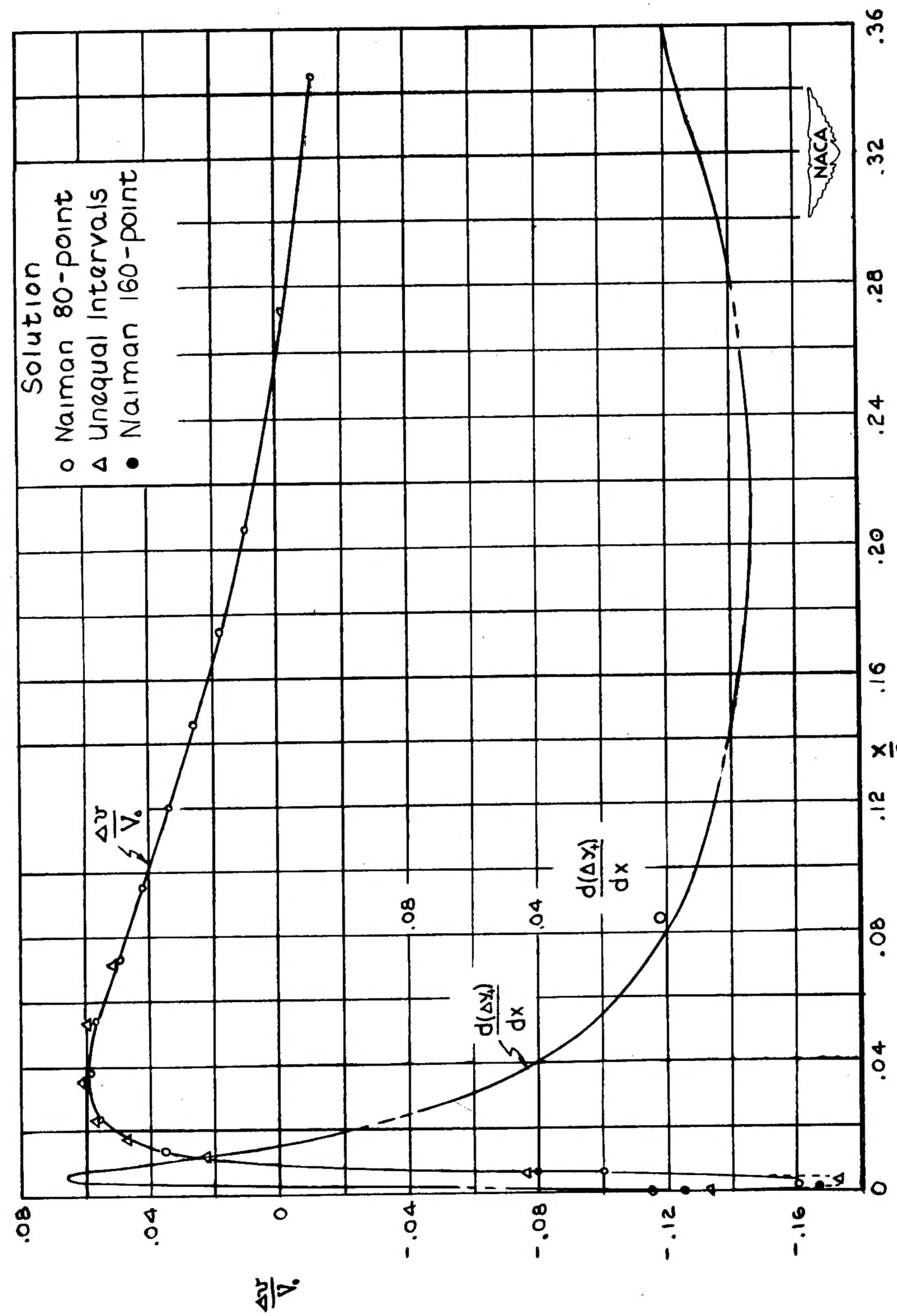


Figure 16.- Results obtained for example II by method of unequal intervals compared with results obtained by Naiman's 80- and 160-point solutions.

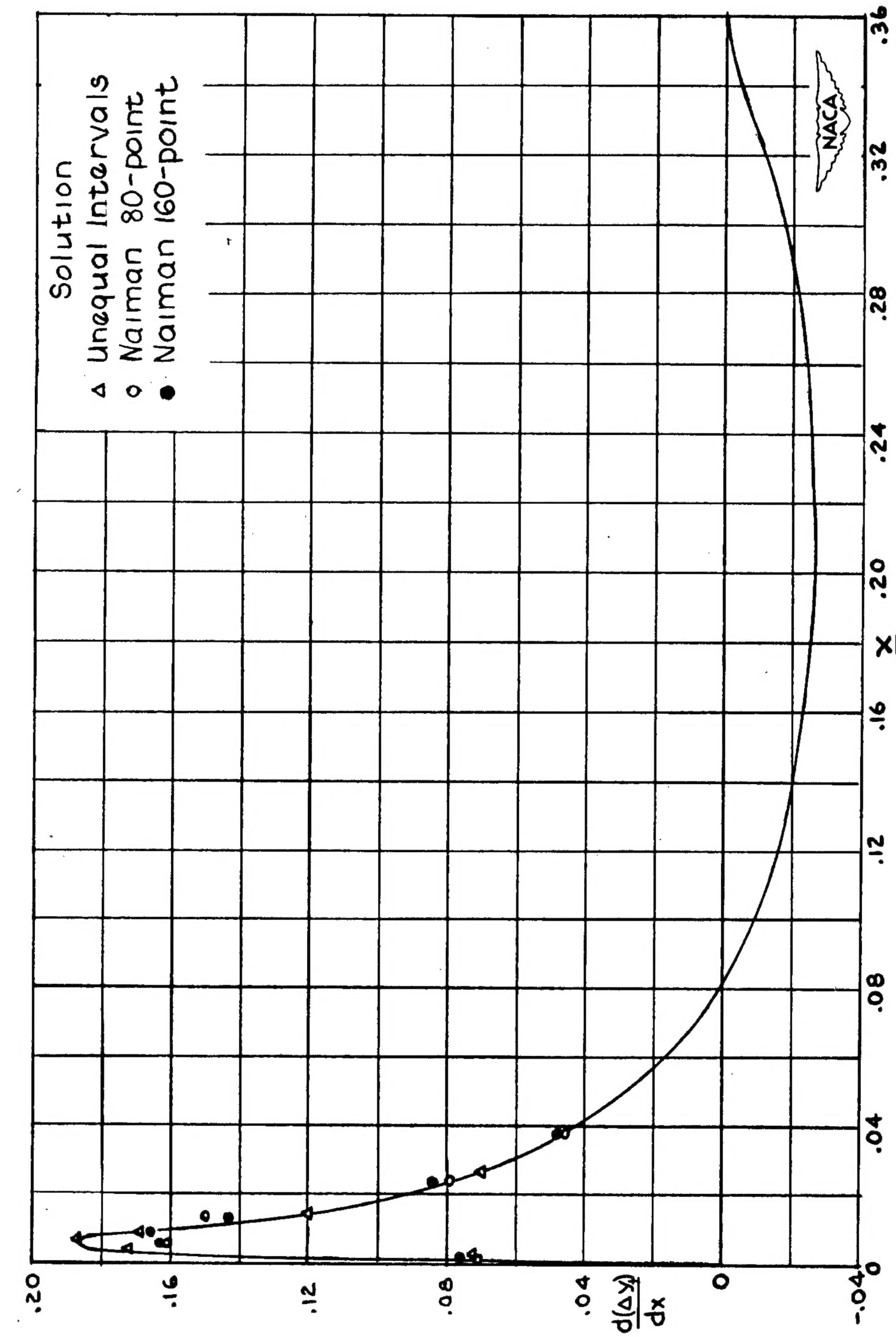


Figure 17.- Solution of inverse problem. Results obtained for example II by method of unequal intervals compared with results obtained by Naiman's 80- and 160-point solutions.

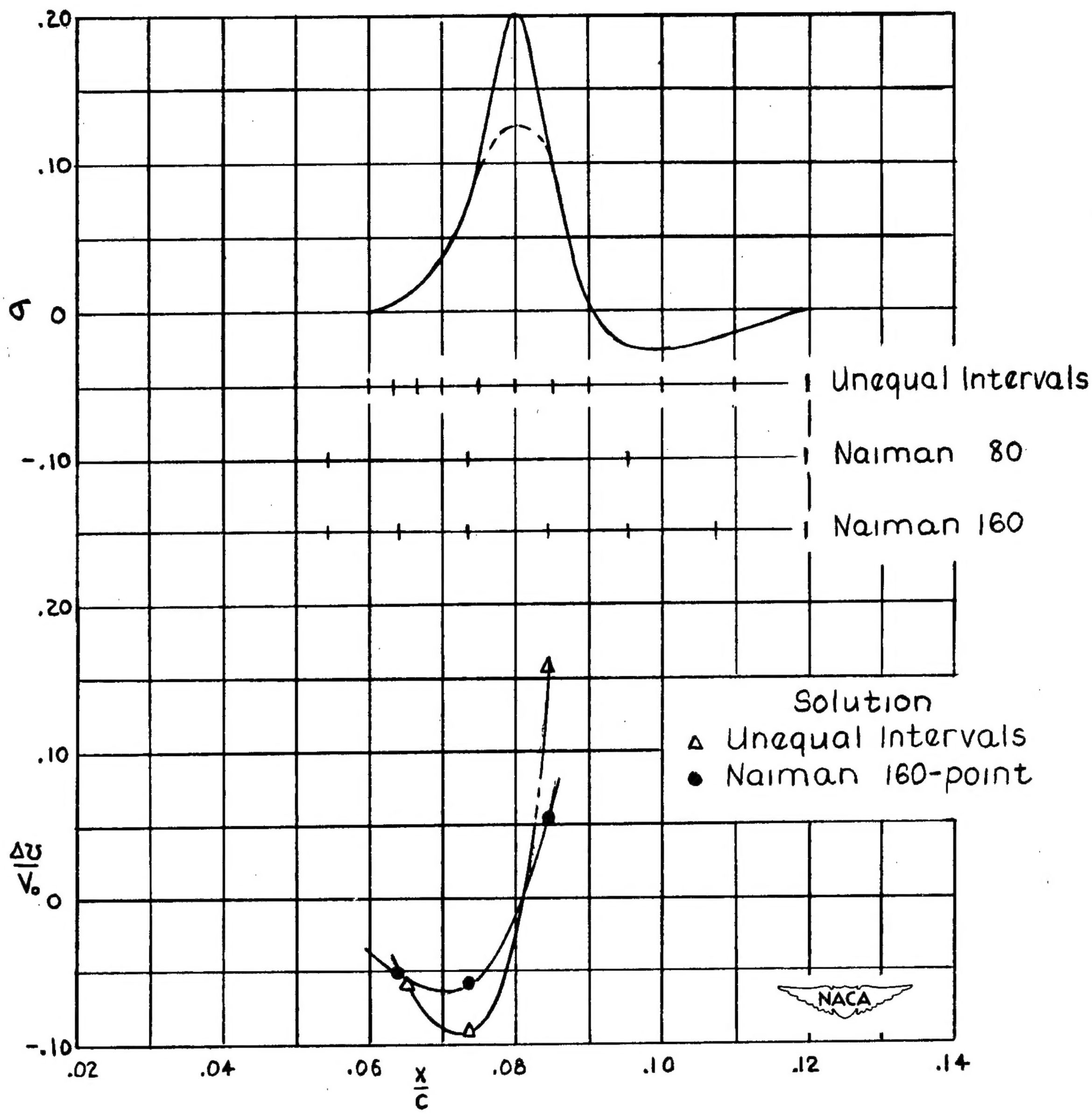


Figure 18.- Results obtained for example III by method of unequal intervals compared with results obtained by Naiman's 80- and 160-point solutions.

NACA TN 2451
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MATHEMATICAL IMPROVEMENT OF METHOD FOR COMPUTING POISSON INTEGRALS INVOLVED IN DETERMINATION OF VELOCITY DISTRIBUTION ON AIRFOILS. I. Flügge-Lotz, Stanford University. October 1951. 84p. diagrs., 3 tabs. (NACA TN 2451)

A solution is presented of the Poisson integral involved in the determination of the change in velocity distribution resulting from a change in airfoil profile in parallel incompressible flow. Three well developed numerical methods of evaluating this integral, all based on the division of the range of integration into small equal intervals, and the difficulties involved in each method, are discussed. A new method based on the use of unequal intervals, is developed and compared with the other methods by means of several examples.

Copies obtainable from NACA, Washington

1. Flow, Incompressible (1.1.1)
2. Wing Section Theory (1.2.1.1)
3. Research Technique, Mathematics (9.2.7)

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